PART THREE: EPICYCLES

1) A LITTLE HISTORICAL SURVEY

About 140 AD the Greek mathematician and astronomer **Claudius Ptolemy** of Alexandria (in Egypt) wrote a book whose original title was "Mathematical Treatise" but has been famous since the Middle Ages as the "Almagest" (i.e. "The very great" in Arabic) in which he developed the whole Babylonian and Greek astronomical knowledge of his time. This book, which became the "bible", first for Ancient Greek and Roman, and afterwards also for Arabic (Islamic) and European astronomy for about 14 centuries (no progress worthy of note was made in astronomy during these centuries…), was based on **three** important "physical" **principles** defined five centuries earlier by the Greek philosopher **Aristotle** (385 – 322 BC):

1st principle: geocentrism

The Earth is the motionless centre of the Universe around which move all celestial bodies, in particular the Sun and the planets. This hypothesis was questioned very early by the Greek astronomer **Aristarchus of Samos** (ca 310 BC – 230 BC) who tried to prove that the Earth is turning around an axis and moving around the Sun in a circular motion but his arguments were not strong enough to convince a lot of people at the time…

2nd principle: dichotomy

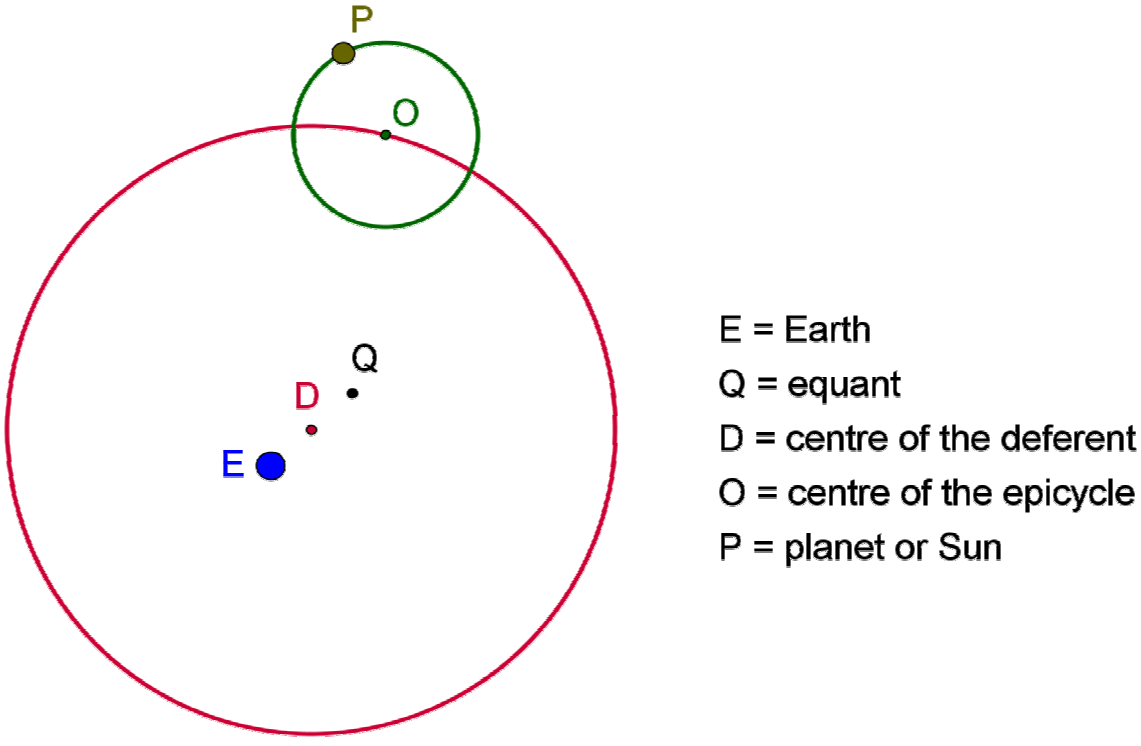
The Universe is divided in two clearly separated parts: on the one hand the "**sublunary**" sphere (i.e. the space between the Earth and the Moon) where all bodies are composed of one or all of the four classical elements (earth, water, air, fire), are perishable and submitted only to up-and down-ward locomotion (downward for the heavy bodies, earth and water, upward for the light bodies, air and fire), on the other hand the "**super-lunary"** sphere (i.e. the space beyond the Moon) where all bodies are made of one matter, the imperishable "aether", so they are not subject

to generation and corruption, their motions are eternal and absolutely perfect and the terrestrial physical laws are not any more valid in this sphere.

3rd principle : uniform circular motion

The only motion worthy of celestial bodies in this super-lunary sphere is the uniform circular motion or, failing that, a combination of such motions.

Now even the earliest observations (long before Ptolemy and Aristotle!) clearly showed that the distances Earth-planets and Earth-Sun were variable, that the apparent motions of the planets had some accelerations, stagnations and even retrograde motions! In order to explain these "anomalies" without rejecting the Aristotelian principles the Greek astronomer **Apollonius of Perga** invented the geometrical model of "**epicycles**" which was taken up and developed three centuries later by Ptolemy. According to this model each planet P (and the Sun!) has a uniform circular motion whose centre O has itself a uniform circular motion whose centre D is the midpoint of the Earth (E) and a point Q called the "**equant**" (i.e. the point from which you can observe the planet P moving with a *constant angular speed*) . The circle of centre D is called the "**deferent**".



By introducing that eccentric model (the Earth was not any more equal to the equant as thought Aristotle and hence not the centre of the deferent) Ptolemy could make very precise calculations about the motion of planets and his successors didn't introduce any fundamental changes to that theory, just some minor improvements… until the Renaissance.

Let's give some short indications of the main steps that led to the final abandon of the Ptolemaic system:

- In 1543, just before his death, the Polish monk **Nicolaus Copernicus** (1473-1543) published his very famous work written years before "*De revolutionibus orbium caelestium"* (*On the Revolutions of the Celestial Spheres*) in which he abandoned the geocentrism for the **heliocentrism** ("helios" means "sun" in ancient Greek). In this book he actually gave the first comprehensive description of the Universe with the Sun at its centre, all planets, inclusively the Earth, turning around the Sun. The theory was still purely geometrical but this change of coordinate system called "Copernican revolution" is one of the most important revolutions in science and yet he kept the other two principles of Aristotelian physics, especially the circular motion of the planets.
- One of the first great supporters of this new theory was the Danish astronomer **Tycho Brahe** (1546-1601) who improved a lot the instruments of observation and who built the first scientific observatories of modern times, first on a Danish island (Uraniborg), then near Prague in Bohemia where he had a brilliant young assistant, Johannes Kepler.
- At the death of Brahe in 1601, **Johannes Kepler** (1571-1630) inherited his huge collection of observations which made it possible to him to develop his own theories. In 1619 he could establish his three laws concerning the motion of planets and which are still valid nowadays, in particular that one which says that the path of a planet is not circular but elliptic and that the Sun is situated on one focus of the ellipse.
- Another very important supporter of the Copernican cosmology was the Italian **Galileo Galilei** (1564-1642) who was the greatest physicist of these times and who is considered to be the founder of modern physics. He was the first one to

understand that the telescope, a Dutch invention of the beginning of the 17th century, could be employed for astronomical observation after having improved it. He asserted that the Copernican system was not only a geometrical artefact but a physical reality! In the very sharp conflict he had therefore with the Catholic Church he unfortunately was not able to give a correct prove of his assertion because for some obscure reasons he refused to recognize the accuracy of Keplers ideas.

- The final point of this evolution was reached in 1687 with the publication by the English physicist **Isaac Newton** (1642-1727) of his "*Philosophiae naturalis principia mathematica"* ("Mathematical principles of naturel philosophy"). In this work, using the new methods of calculus developed by himself (derivatives, integrals, etc), he gave a mathematical proof of Kepler's laws from the fundamental formula of physics $F = m\gamma$ (also established by himself), in other words he showed that the same physical laws rule the sublunarian and the super-lunarian world and that this distinction was now completely useless.

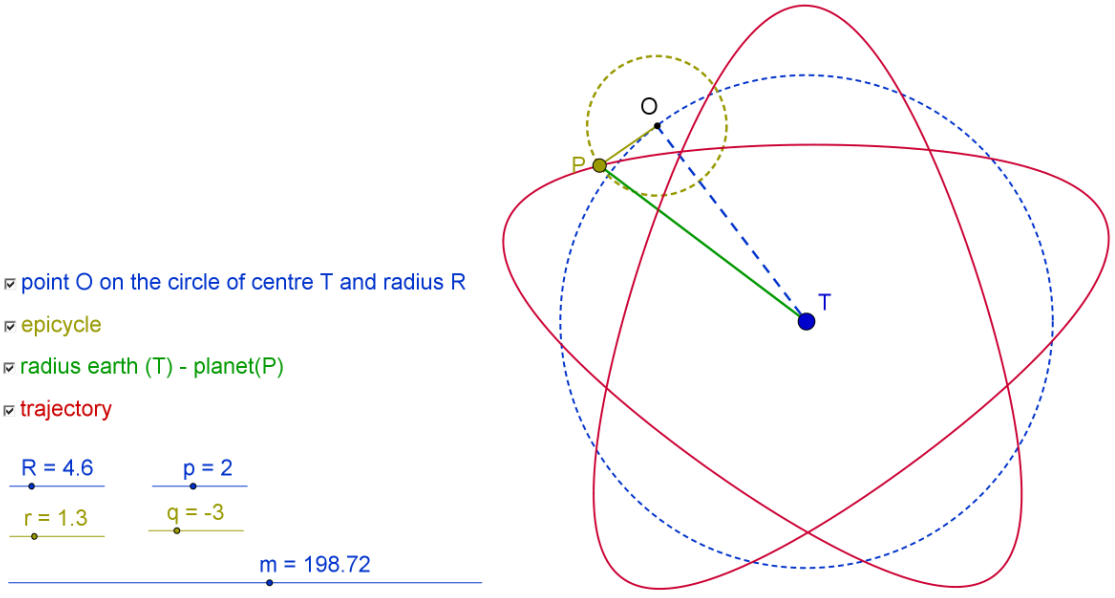
2) CONSTRUCTION OF EPICYCLES

In order to show the richness of the theory of epicycles by the great variety of curves you get out of it, we shall now realize a construction with GeoGebra to visualize the trajectory of a point P (planet or sun) situated on an epicycle of centre O turning around a fixed point T (earth), without considering eccentricity.

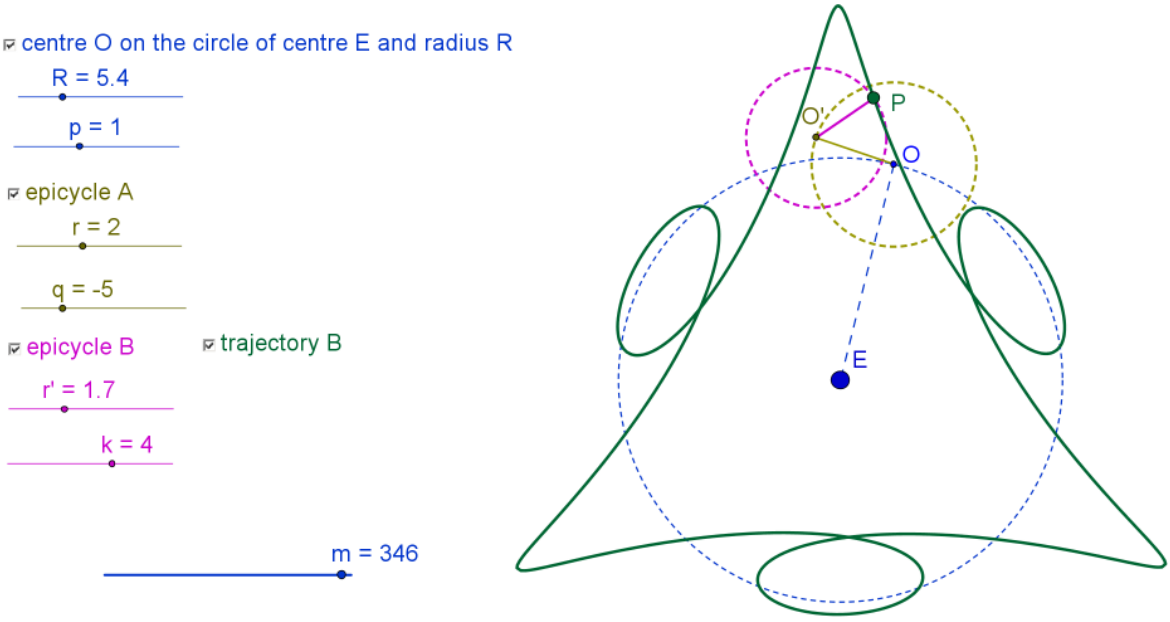
For that you can do the following (or find your own construction!):

- Define a slider R (e.g. $0 \le R \le 15$) and a point T.
- Trace a circle of centre T and (variable) radius R.
- Trace a fixed point A on this circle (in the uppermost position).
- Define a slider m with $0 \le m \le 360$ and increment 0,1.
- Define a slider p such that **p** is an **integer** (e.g. $-15 \le p \le 15$ and increment 1).
- Construct the image O of A by the rotation around T by angle $p \cdot m°$: p indicates the number of complete revolutions O does when m varies from $0°$ to $360°$, in the positive "direction" if $p > 0$, in the opposite direction else.

- Define a slider r (e.g. $0 \le r \le 10$) and trace the circle of centre O and (variable) radius r: this circle is the epicycle.
- Trace a fixed point B on this circle (in the uppermost position).
- Define a slider q such that **q** is an **integer** (e.g. $-15 \le p \le 15$ and increment 1).
- Construct the image P of B by the rotation around O by angle $q \cdot m°$: q indicates the number of complete revolutions P does when m varies from $0°$ to $360°$, in the positive "direction" if $q > 0$, in the opposite direction else.
- Hide the points A and B.
- Draw the segments $[TO]$ and $[OP]$: they allow a better observation (use the command "*animation on*" by right-click on slider m) of the movement of O around T and P around O (constant angular speed!).
- Draw also the segment $[TP]$ in order to be able to compare the angular speed of O and P turning around T, i.e. the apparent speed of the planet P "turning" around T!
- For a better visualisation of the curves described by P when m varies from $0°$ to $360°$ use the "**locus**" command.
- Example: $p = 2$ et $q = -3$ (i.e. During the time O makes 2 complete revolutions around T, P turns 3 times around O in the opposite direction !



The amateurs of really strange curves may try to construct “epicycles of epicycles” where the planet P turns around O’ who turns around O who turns around the Earth as shown in the following figure:



On this example, during the time O turns once anticlockwise around E, O’ turns five times clockwise around O and P four times anticlockwise around O’ but you can try a lot of other combinations!