Nom : .... Prénom : ....

2B1 Corrigé du devoir I,2 7.12.12

Question 1

(1) $f : x \mapsto \dfrac{x^2+2x-3}{|x+2|}$

a) $D_f = D_c f = \mathbb{R} \setminus \{-2\}$

b) • $\lim\limits_{x \to -2} f(x) = \text{"}\dfrac{4-4-3}{0^+}\text{"} = -\infty$

$\Rightarrow$ A.V. : $x = -2$

• $\lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{x \to \pm\infty} \dfrac{x^2(\overbrace{1+\frac{2}{x}-\frac{3}{x^2}}^{\to 1})}{|x| \cdot \underbrace{|1+\frac{2}{x}|}_{\to 1}}$

$= \lim\limits_{x \to \pm\infty} |x| = +\infty$

$\Rightarrow$ pas d'A.H.

• $\lim\limits_{x \to \pm\infty} \dfrac{f(x)}{x} = \lim\limits_{x \to \pm\infty} \dfrac{|x|}{x} = \lim\limits_{x \to \pm\infty} \dfrac{\pm \cancel{x}}{\cancel{x}} = \pm 1$

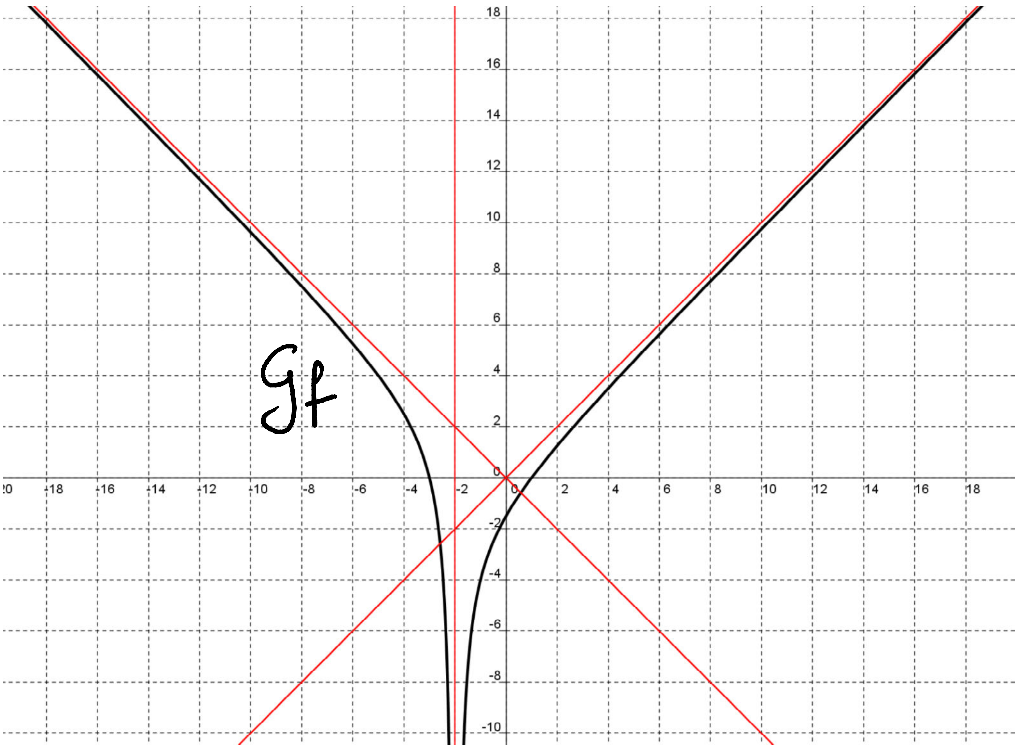
• $\lim\limits_{x \to \pm\infty} f(x) \mp x = \lim\limits_{x \to \pm\infty} \dfrac{x^2+2x-3}{\pm(x+2)} \mp x$

$= \lim\limits_{x \to \pm\infty} \pm \dfrac{x^2+2x-3-x(x+2)}{x+2}$

$= \lim\limits_{x \to \pm\infty} \pm \dfrac{\cancel{2x}-3-\cancel{2x}}{x+2} = \text{"}\dfrac{\mp 3}{\pm\infty}\text{"} = 0^-$

$\to$ A.O.D : $y = x$ et A.O.G : $y = -x$

(et G_f est situé en-dessous des 2 A.O. au voisin. de $\pm\infty$)



(2) $g : x \mapsto \sqrt{\dfrac{4x^2-1}{x^2+x-2}}$

C.E : $\dfrac{4x^2-1}{x^2+x-2} \geqslant 0$

x	$-\infty$		-2		$-\frac{1}{2}$		$\frac{1}{2}$		1		$+\infty$
$4x^2-1$		$+$	|	$+$	0	$-$	0	$+$	|	$+$	
x^2+x-2		$+$	0	$-$	|	$-$	|	$-$	0	$+$	
$\frac{4x^2-1}{x^2+x-2}$		$+$	||	$-$	0	$+$	0	$-$	||	$+$	

a) $D_g = D_c g =]-\infty;-2[\cup [-\frac{1}{2};\frac{1}{2}] \cup]1,+\infty[$

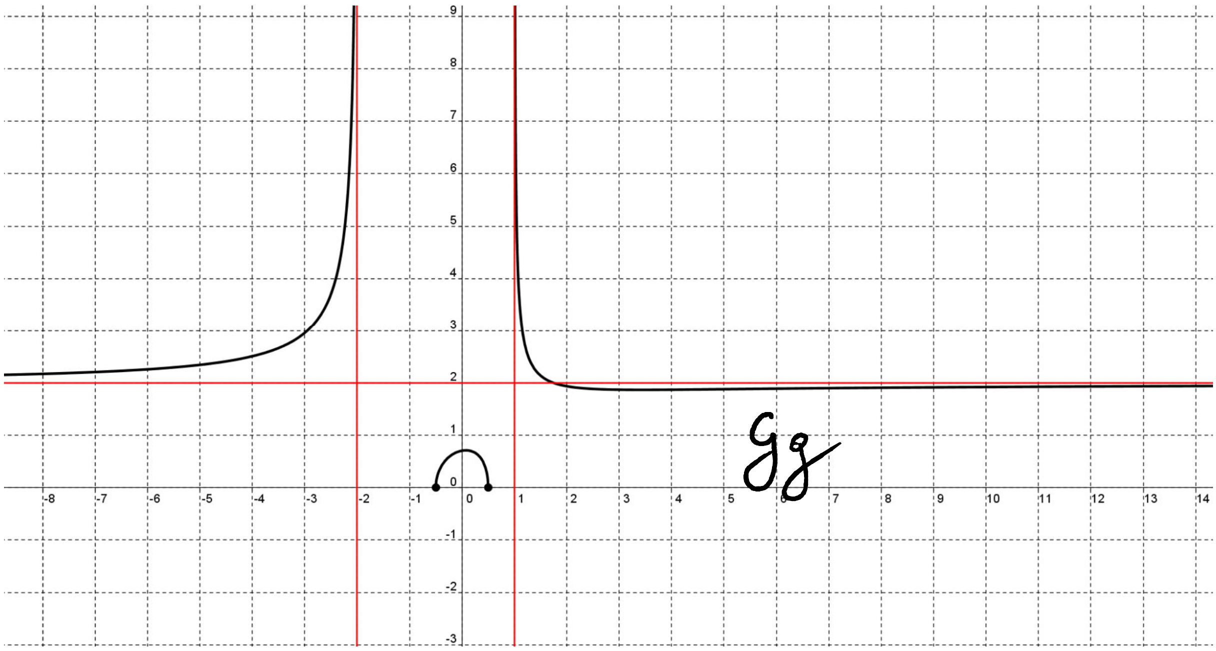
b) • $\lim\limits_{x\to 1^{(+)}} g(x) = "\sqrt{\frac{3}{0^+}}" = +\infty \Rightarrow$ A.V : $x=1$

• $\lim\limits_{x\to -2^{(-)}} g(x) = "\sqrt{\frac{15}{0^+}}" = +\infty \Rightarrow$ A.V. : $x=-2$

• $\lim\limits_{x\to -\frac{1}{2}^{(+)}} g(x) = g(-\frac{1}{2}) = 0 \Rightarrow g$ cont. (à droite) en $-\frac{1}{2}$.

• $\lim\limits_{x\to \frac{1}{2}^{(-)}} g(x) = g(\frac{1}{2}) = 0 \Rightarrow g$ cont. (à gauche) en $\frac{1}{2}$.

• $\lim\limits_{x\to\pm\infty} g(x) = \lim\limits_{x\to\pm\infty} \sqrt{\frac{4\cancel{x^2}}{\cancel{x^2}}} = \sqrt{4} = 2 \Rightarrow$ A.H. : $y=2$



(3) $\quad p : x \mapsto x\sqrt{\dfrac{x-1}{x+2}}$

a) $D_p = D_c p = \left]-\infty\,; -2\right[\cup \left[1, +\infty\right[$

b) • $\lim\limits_{x \to -2^{(-)}} p(x) = \text{"}-2 \cdot \sqrt{\dfrac{-3}{0^-}}\text{"} = -\infty \Rightarrow$ A.V. : $x = -2$.

• $\lim\limits_{x \to 1^+} p(x) = 0 = p(1) \Rightarrow p$ est cont. (à droite) en 1.

• $\lim\limits_{x \to \pm\infty} p(x) = \lim\limits_{x \to \pm\infty} x \cdot \underbrace{\sqrt{\dfrac{x-1}{x+2}}}_{\to 1} = \pm\infty \Rightarrow$ pas d'A.H.

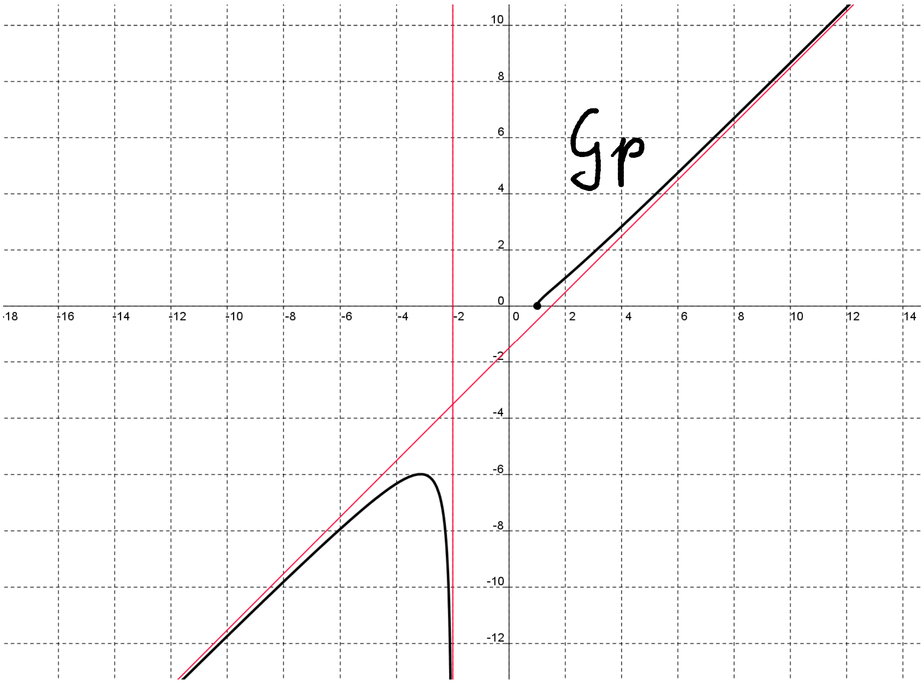
• $\lim\limits_{x \to \pm\infty} \dfrac{p(x)}{x} = \lim\limits_{x \to \pm\infty} \sqrt{\dfrac{x-1}{x+2}} = 1 \quad (= a)$

• $\lim\limits_{x \to \pm\infty} \left(p(x) - x\right) = \lim\limits_{x \to \pm\infty} x\left(\sqrt{\dfrac{x-1}{x+2}} - 1\right) \quad$ (f.i. $\infty \cdot 0$)

$$= \lim_{x \to \pm\infty} x \cdot \frac{\frac{x-1}{x+2} - 1}{\underbrace{\sqrt{\frac{x-1}{x+2}}}_{\to 1} + 1}$$

$$= \lim_{x \to \pm\infty} \frac{x}{2} \cdot \frac{x-1-(x+2)}{x+2} = \lim_{x \to \pm\infty} \frac{-3\cancel{x}}{2\cancel{x}} = -\frac{3}{2}$$

$\Rightarrow$ A.O : $y = x - \frac{3}{2}$



(4) $\quad q : x \mapsto \dfrac{x^2-4}{\sqrt{x^2-2x}}$

a) $D_q = D_c q = \left]-\infty\,;0\right[\cup \left]2\,,+\infty\right[$

b) • $\lim\limits_{x\to 0^{(-)}} q(x) = "\dfrac{-4}{\sqrt{0^+}}" = -\infty \Rightarrow$ A.V. : $x=0$

• $\lim\limits_{x\to 2^+} q(x) = \lim\limits_{x\to 2^+} \dfrac{(x-2)(x+2)}{\sqrt{x(x-2)}}$ $\quad$ (fi "$\frac{0}{0}$")

$= \lim\limits_{x\to 2^+} \dfrac{\sqrt{x-2}\cdot(x+2)}{\sqrt{x}} = \dfrac{0^+\cdot 4}{\sqrt{2}} = 0$

$\Rightarrow$ G_q admet un trou en $(2,0)$

• $\lim\limits_{x\to\pm\infty} q(x) = \lim\limits_{x\to\pm\infty} \dfrac{x^2}{\sqrt{x^2}} = \lim\limits_{x\to\pm\infty} \dfrac{x^2}{|x|}$

$= \lim\limits_{x\to\pm\infty} |x| = +\infty \Rightarrow$ pas d'A.H.

• $\lim\limits_{x\to\pm\infty} \dfrac{q(x)}{x} = \lim\limits_{x\to\pm\infty} \dfrac{x^2}{x|x|} = \lim\limits_{x\to\pm\infty} \dfrac{x}{|x|} = \pm 1$

• $\lim\limits_{x\to\pm\infty} q(x) \mp x = \lim\limits_{x\to+\infty} \dfrac{x^2-4 \mp x\sqrt{x^2-2x}}{\sqrt{x^2-2x}}$

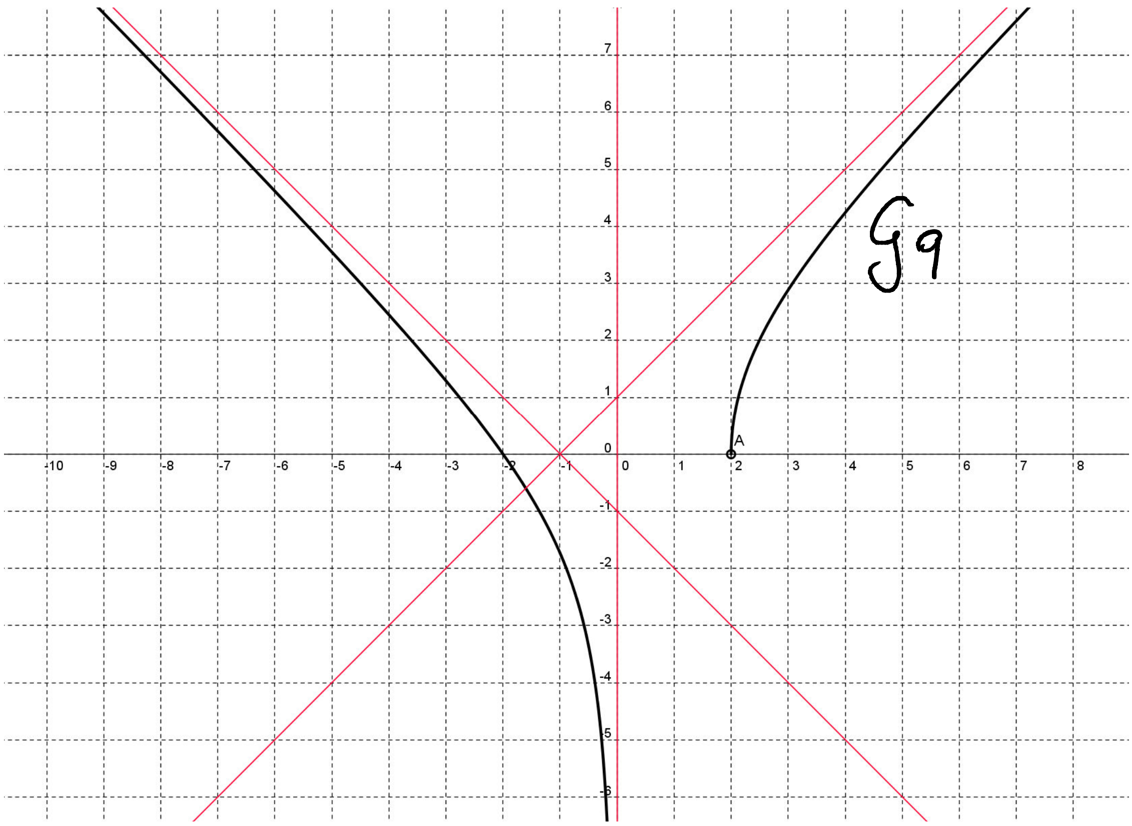
$$= \lim_{x\to\pm\infty} \frac{(x^2-4)^2 - x^2(x^2-2x)}{|x|\underbrace{\sqrt{1-\frac{2}{x}}}_{\to 1}\cdot\left(x^2-4 \pm x\sqrt{x^2-2x}\right)}$$

$$= \lim_{x\to\pm\infty} \frac{\cancel{x^4} - 8x^2 + 16 - \cancel{x^4} + 2x^3}{|x|\cdot x^2\left(1 - \frac{4}{x^2} \pm \frac{\cancel{x}|x|}{x^{\cancel{2}}}\cdot\sqrt{1-\frac{2}{x}}\right)}$$

$$= \lim_{x\to\pm\infty} \frac{2x^3}{|x|\cdot x^2\left(1 \pm \underbrace{\frac{|x|}{x}}_{=\pm 1}\cdot\underbrace{\sqrt{1-\frac{2}{x}}}_{\to 1}\right)}$$

$$= \lim_{x\to\pm\infty} \frac{2\cancel{x^3}}{\pm 2\cancel{x^3}} = \pm 1$$

$\Longrightarrow$ A.O.D: $y = x+1$ et A.O.G: $y = -x-1$



G. Lorang