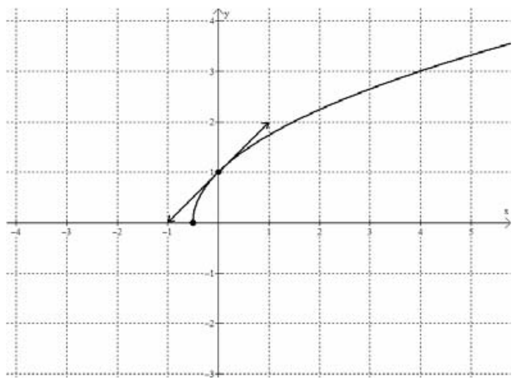
Nom : …………………………… Prénom : …………………………..

2D Corrigé du devoir de mathématiques I,3 9.12.08

Question 1

(1) $f(x) = \sqrt{2x+1}$ $a = 0$;

$$f'(0) = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$$
$$= \lim_{x\to 0}\frac{\sqrt{2x+1}-1}{x}$$
$$= \lim_{x\to 0}\frac{\left(\sqrt{2x+1}-1\right)\left(\sqrt{2x+1}+1\right)}{x\left(\sqrt{2x+1}+1\right)}$$
$$= \lim_{x\to 0}\frac{2x+1-1}{x\left(\sqrt{2x+1}+1\right)}$$
$$= \lim_{x\to 0}\frac{2x}{x\left(\sqrt{2x+1}+1\right)}$$
$$= \lim_{x\to 0}\frac{2}{\sqrt{2x+1}+1}$$
$$= \frac{2}{2} = 1$$



I.G. : La tangente à $\mathcal{G}_f$ au point (0,1) a comme pente 1.

(2) $f(x) = 1+2x-3x^2$ $a = 1$;

$$f'(1) = \lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$$
$$= \lim_{x\to 1}\frac{1+2x-3x^2-0}{x-1}$$
$$= \lim_{x\to 1}\frac{-3\cancel{(x-1)}\left(x+\frac{1}{3}\right)}{\cancel{x-1}}$$
$$= \lim_{x\to 1} -3\left(x+\tfrac{1}{3}\right)$$
$$= -3\,\tfrac{4}{3} = -4$$

I.G. : La tangente à $\mathcal{G}_f$ au point (1,0) a comme pente –4.

Question 2

(1) $f(x) = x^3 - 8x^2 - 5x + 1$;

$f'(x) = 3x^2 - 16x - 5$;

$\operatorname{dom} f = \operatorname{dom} f' = \mathbb{R}$.

(2) $f(x) = \dfrac{\sqrt{x}}{2} - \dfrac{1}{\sqrt{x}} = \dfrac{1}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}}$;

$f'(x) = \dfrac{1}{4}x^{-\frac{1}{2}} + \dfrac{1}{2}x^{-\frac{3}{2}} = \dfrac{1}{4\sqrt{x}} + \dfrac{1}{2x\sqrt{x}}$;

$\operatorname{dom} f = \operatorname{dom} f' = \mathbb{R}_+^*$.

(3) $f(x) = 3x^2\sqrt{x} - \dfrac{5}{2x} = 3x^{\frac{5}{2}} - \dfrac{5}{2}x^{-1}$;

$f'(x) = \dfrac{15}{2}x^{\frac{3}{2}} + \dfrac{5}{2}x^{-2} = \dfrac{15x\sqrt{x}}{2} + \dfrac{5}{2x^2}$;

$\operatorname{dom} f = \operatorname{dom} f' = \mathbb{R}_+^*$.

(4) $f(x) = \dfrac{2x^2 - 7}{x+1}$;

$$\begin{aligned} f'(x) &= \frac{2x^2-7}{x+1} \\ &= \frac{4x(x+1) - \left(2x^2-7\right)}{(x+1)^2} \\ &= \frac{2x^2+4x+7}{(x+1)^2} \end{aligned}$$

$\operatorname{dom} f = \operatorname{dom} f' = \mathbb{R}\backslash\{-1\}$

(5) $f(x) = (3x+1)\sqrt{x}$;

$$\begin{aligned} f'(x) &= 3\sqrt{x} + (3x+1)\cdot\frac{1}{2\sqrt{x}} \\ &= \frac{6x}{2\sqrt{x}} + \frac{3x+1}{2\sqrt{x}} \\ &= \frac{9x+1}{2\sqrt{x}} \end{aligned}$$

$\operatorname{dom} f = \mathbb{R}_+$; $\operatorname{dom} f' = \mathbb{R}_+^*$.

Question 3

(1) $\operatorname{dom} h = \mathbb{R}^*$.

(2) Il est clair que h est continue en tout réel distinct de 1 et de 0. De plus, h est continue à droite en 1 car :

$$\lim_{x\to 1^+} h(x) = \lim_{x\to 1^+} x^2 = 1 = h(1)$$

Pour que h soit continue en 1 il faut et il suffit donc que :

$$\lim_{x\to 1^-} h(x) = 1.$$

Or, $\displaystyle\lim_{x\to 1^-} h(x) = a - \frac{1}{1} = a - 1$. Donc : $a - 1 = 1 \Leftrightarrow a = 2$.

(3) On a :

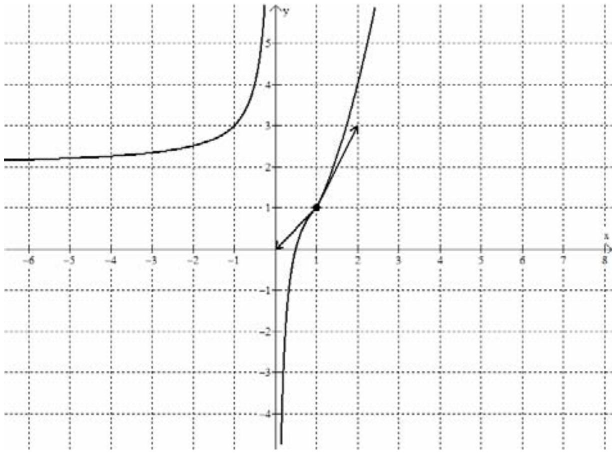
$$\begin{aligned} h_d'(1) &= \lim_{x\to 1^+} \frac{h(x)-h(1)}{x-1} \\ &= \lim_{x\to 1^+} \frac{x^2-1}{x-1} \\ &= \lim_{x\to 1^+} x+1 \\ &= 2 \end{aligned}$$

et

$$\begin{aligned} h_g'(1) &= \lim_{x\to 1^-} \frac{h(x)-h(1)}{x-1} \\ &= \lim_{x\to 1^-} \frac{2-\frac{1}{x}-1}{x-1} \\ &= \lim_{x\to 1^-} \frac{1-\frac{1}{x}}{x-1} \\ &= \lim_{x\to 1^-} \frac{\frac{x-1}{x}}{x-1} = \lim_{x\to 1^-} \frac{1}{x} = 1 \end{aligned}$$

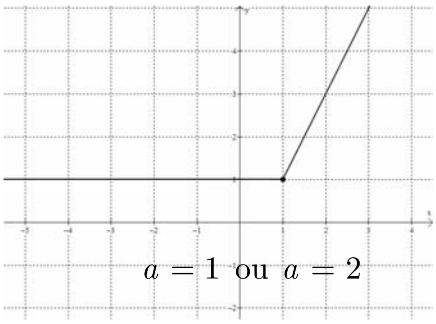
Donc h n'est pas dérivable en 1 car $h_d'(1) \neq h_g'(1)$.

Le point (1,1) du graphe de h est un ***point anguleux***.



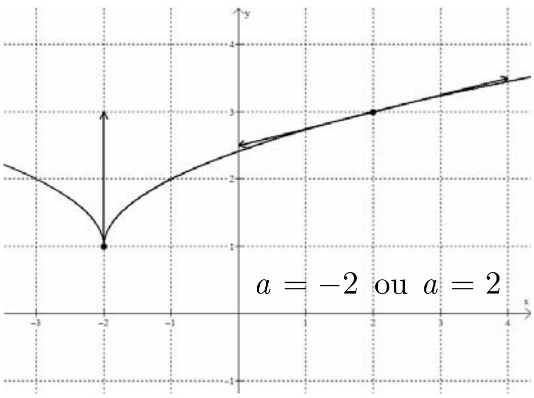
Question 4

(1)



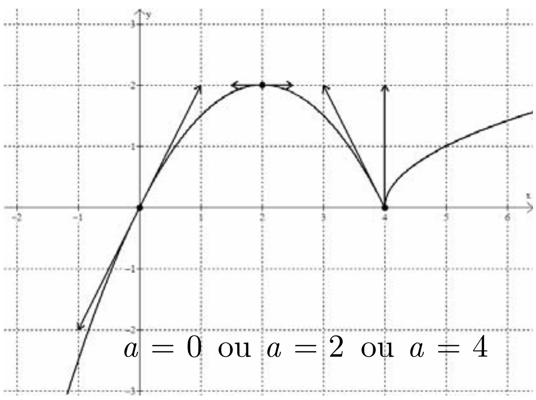
a	1	2
$f(a)$	1	3
$f'(a)$	/	2
$f_d'(a)$	2	2
$f_g'(a)$	0	2

(2)



a	–2	2
$f(a)$	1	3
$f'(a)$	/	$\frac{1}{4}$
$f_d'(a)$	/	$\frac{1}{4}$
$f_g'(a)$	/	$\frac{1}{4}$

(3)



a	0	2	4
$f(a)$	0	2	0
$f'(a)$	2	0	/
$f_d'(a)$	2	0	/
$f_g'(a)$	2	0	–2

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