3B1 correction du devoir II, 1

Question 1

Voir manuel

Question 2

(1) a) $\sin\left(-\frac{108\pi}{7}\right) = \sin\left(-15\pi - \frac{3\pi}{7}\right)$

$= -\sin\left(-\frac{3\pi}{7}\right)$

$= \sin\left(\frac{3\pi}{7}\right)$

$= \cos\left(\frac{\pi}{2} - \frac{3\pi}{7}\right)$

$= \cos\left(\frac{\pi}{14}\right)$

b) $\cos\left(\frac{48\pi}{5}\right) = \cos\left(9\pi + \frac{3\pi}{5}\right)$

$= -\cos\left(\frac{3\pi}{5}\right)$

$= -\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)$

$= -\sin\left(-\frac{\pi}{10}\right)$

$= \sin\left(\frac{\pi}{10}\right)$

c) $\tan\left(-\frac{89\pi}{12}\right) = \tan\left(-7\pi - \frac{5\pi}{12}\right)$

$= -\tan\left(\frac{5\pi}{12}\right)$

$= -\cot\left(\frac{\pi}{2} - \frac{5\pi}{12}\right)$

$= -\cot\left(\frac{\pi}{12}\right)$

(2) $\cot(x+3\pi)\cos\left(x - \frac{3\pi}{2}\right) - \dfrac{\cos\left(x+\frac{5\pi}{2}\right)}{\tan\left(x-\frac{3\pi}{2}\right)}$

$= -\cot x \cdot \cos\left(x - \frac{\pi}{2}\right) - \dfrac{\cos(x+\pi/2)}{\tan\left(x-\frac{\pi}{2}\right)}$

$= -\cot x \cdot \cos\left(\frac{\pi}{2} - x\right) - \left(\dfrac{-\sin x}{-\tan\left(\frac{\pi}{2}-x\right)}\right)$

$= -\dfrac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x} - \dfrac{\sin x}{\cot x}$

$= -\cos x - \dfrac{\sin^2 x}{\cos x} = -\dfrac{\cos^2 x + \sin^2 x}{\cos x} = -\dfrac{1}{\cos x}$

Question 3

a) $\cot\alpha = \frac{4}{3} \Rightarrow \tan\alpha = \frac{3}{4}$

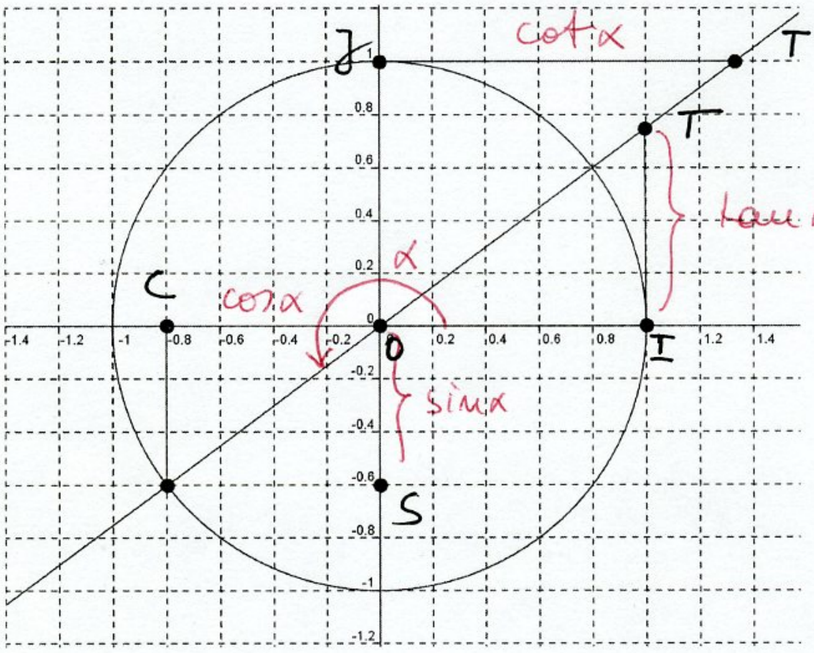
$\cos^2\alpha = \frac{1}{1+\tan^2\alpha} = \frac{1}{1+ 9/16} = \frac{1}{25/16} = \frac{16}{25}$

$\Rightarrow \cos\alpha = -\frac{4}{5}$ car dans le 3^e q. $\cos\alpha < 0$.

$\sin^2\alpha = 1 - \cos^2\alpha = 1 - \frac{16}{25} = \frac{9}{25}$

$\Rightarrow \sin\alpha = -\frac{3}{5}$ car dans le 3^e q. $\sin\alpha < 0$.

b)



c) $\alpha = \tan^{-1}\left(\frac{3}{4}\right) + \pi \simeq 3{,}7851$ rad.

Mesure principale : $3{,}7851 - 2\pi \simeq -2{,}4981$ rad

Question 4

(1) Posons $\cos^2 x = y$. Alors l'équation devient :

$y^2 + y - \frac{3}{4} = 0$. $\Delta = 1 + 3 = 4$

$y_1 = \frac{-1-2}{2} = -\frac{3}{2}$ $\qquad y_2 = \frac{-1+2}{2} = \frac{1}{2}$

Revenons à x :

$\cos^2 x = -\frac{3}{2}$ impossible ou $\cos^2 x = \frac{1}{2}$

Donc $\cos^2 x = \frac{1}{2}$

$\Leftrightarrow \quad \cos x = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$ ou $\cos x = -\sqrt{\frac{1}{2}} = -\frac{\sqrt{2}}{2}$

$\Leftrightarrow \quad x = \pm\frac{\pi}{4} + k\cdot 2\pi$ ou $x = \pm\frac{3\pi}{4} + k\cdot 2\pi, \quad k \in \mathbb{Z}$

$S = \left\{ \frac{\pi}{4} + k\cdot\frac{\pi}{2} \,/\, k \in \mathbb{Z} \right\}$

(2) $\quad \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) = -\frac{2}{5}$

$\Leftrightarrow \quad \frac{\pi}{4} - \frac{x}{2} = \underbrace{-0{,}4115168}_{\alpha_1}\ldots + k\cdot 2\pi$

ou

$\frac{\pi}{4} - \frac{x}{2} = \pi - \alpha_1 + k\cdot 2\pi \quad , k \in \mathbb{Z}$

$\Leftrightarrow \quad -\frac{x}{2} = -\frac{\pi}{4} + \alpha_1 + k\cdot 2\pi$

ou

$-\frac{x}{2} = -\frac{\pi}{4} + \pi - \alpha_1 + k\cdot 2\pi$

$\Leftrightarrow \quad x = \frac{\pi}{2} - 2\alpha_1 + k\cdot 4\pi$

ou

$x = \frac{\pi}{2} - 2\pi + 2\alpha_1 + k\cdot 4\pi \quad , k \in \mathbb{Z}$

$\Leftrightarrow \quad x = 2{,}39383 0019\ldots + k\cdot 4\pi$

ou

$x = -5{,}53542267\ldots + k\cdot 4\pi \quad , k \in \mathbb{Z}$

Solutions dans $[-3\pi, 4\pi]$

$$\begin{cases} x_1 = 2{,}3938\ldots \in [0,\pi] \\ x_2 = -5{,}5354\ldots \in [-2\pi, -\pi] \\ x_3 = x_2 + 4\pi = 7{,}0309\ldots \in [2\pi, 3\pi] \end{cases}$$

Question 5

(1) $D_f = \mathbb{R}$

f n'est ni paire, ni impaire.

En effet :

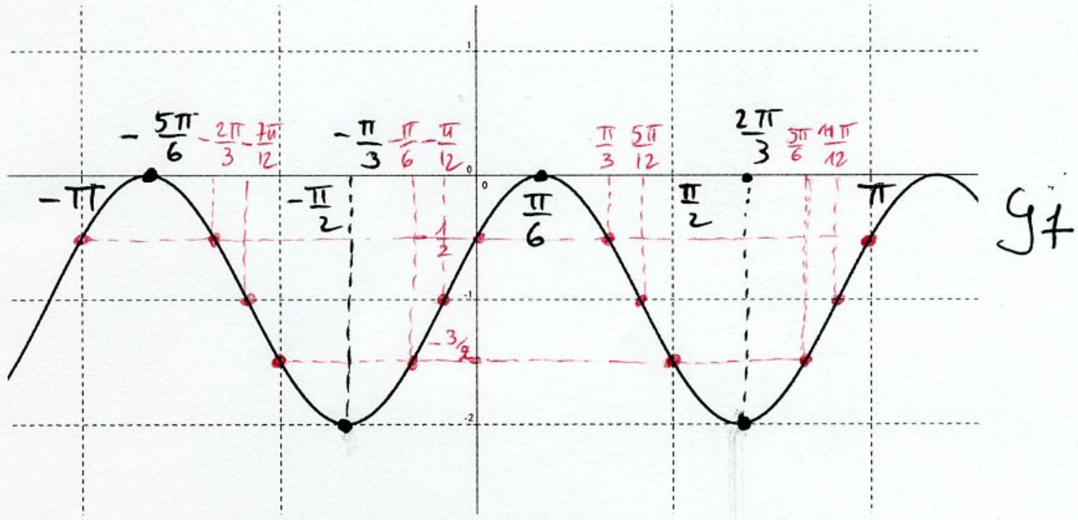
ni opposés
ni égaux

$$f\left(\frac{\pi}{6}\right) = \cos\left(2\cdot\frac{\pi}{6} - \frac{\pi}{3}\right) - 1 = \cos 0 - 1 = 0$$

$$f\left(-\frac{\pi}{6}\right) = \cos\left(-\frac{\pi}{3} - \frac{\pi}{3}\right) - 1 = \cos\left(-\frac{2\pi}{3}\right) - 1 = -\frac{3}{2}$$

f est périodique de période $\frac{2\pi}{2} = \underline{\underline{\pi}}$

(2)



On part de

$\cos : x \longmapsto \cos x$

$f_1 : x \longmapsto \cos\left(x - \frac{\pi}{3}\right)$

$f_2 : x \longmapsto \cos\left(2x - \frac{\pi}{3}\right)$

$f : x \longmapsto \cos\left(2x - \frac{\pi}{3}\right) - 1$

$$\mathcal{G}_{\cos} \xrightarrow[\text{ajouter } \frac{\pi}{3} \text{ aux } x]{} \mathcal{G}_{f_1} \xrightarrow[\text{diviser les } x \text{ par } 2]{} \mathcal{G}_{f_2} \xrightarrow[\text{retrancher } 1 \text{ aux } y]{} \mathcal{G}_f .$$

Question 6

(1) C.E : $\begin{cases} \tan\beta \text{ existe} \Leftrightarrow \cos\beta \neq 0 \\ \cot\beta \text{ existe} \Leftrightarrow \sin\beta \neq 0 \\ \cos\beta \neq 0 \\ \sin\beta \neq 0 \end{cases}$

$\Leftrightarrow \beta \neq k \cdot \frac{\pi}{2} \quad , k \in \mathbb{Z}$

$D = \mathbb{R} \setminus \{ k \cdot \frac{\pi}{2} \ / \ k \in \mathbb{Z} \}$

$$\left(1 + \cot\beta + \frac{1}{\sin\beta}\right)\left(1 + \tan\beta - \frac{1}{\cos\beta}\right)$$

$$= \frac{\sin\beta + \cos\beta + 1}{\sin\beta} \cdot \frac{\cos\beta + \sin\beta - 1}{\cos\beta}$$

$$= \frac{(\cos\beta + \sin\beta + 1)(\cos\beta + \sin\beta - 1)}{\sin\beta \cos\beta}$$

$$= \frac{(\cos\beta + \sin\beta)^2 - 1}{\sin\beta \cos\beta}$$

$$= \frac{\cancel{\cos^2\beta} + 2\cos\beta\sin\beta + \cancel{\sin^2\beta - 1}}{\sin\beta \cos\beta}$$

$$= \frac{2\cancel{\cos\beta}\cancel{\sin\beta}}{\cancel{\sin\beta}\cancel{\cos\beta}} = 2$$

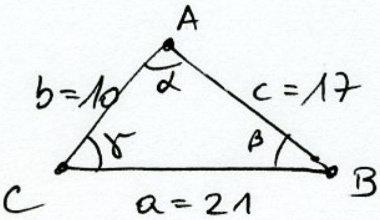
(2) $\sin^4 a - \cos^4 a + 2\cos^2 a$

$= (\sin^2 a - \cos^2 a)\underbrace{(\sin^2 a + \cos^2 a)}_{1} + 2\cos^2 a$

$= \sin^2 a - \cos^2 a + 2\cos^2 a$

$= \sin^2 a + \cos^2 a = \underline{\underline{1}}$

Question 7

(1) 

Théorème d'Al-Kashi :

$$21^2 = 17^2 + 10^2 - 2\cdot 17\cdot 10\cdot \cos\alpha$$

$$\Leftrightarrow 441 = 289 + 100 - 340\cdot\cos\alpha$$

$$\Leftrightarrow \cos\alpha = \frac{389-441}{340} = -\frac{13}{85}$$

$$\sin^2\alpha = 1-\cos^2\alpha = 1-\left(\frac{13}{85}\right)^2 = \frac{7056}{7225}$$

$$\Leftrightarrow \sin\alpha = \frac{84}{85}$$

Aire du ΔABC : $[ABC] = \frac{1}{2}\cdot 10\cdot 17\cdot\sin\alpha$

$$= 5\cdot 17\cdot\frac{84}{85} = 84\text{ cm}^2$$

(2) $\alpha = \cos^{-1}\left(-\frac{13}{85}\right) = 98°\,47'\,50{,}68''$

α est un angle obtus,
donc β et γ sont des angles aigus.

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Leftrightarrow \sin\beta = \frac{b\sin\alpha}{a}$$

$$\Leftrightarrow \sin\beta = \frac{{}^{2}\cancel{10}\cdot\cancel{84}^{4}}{\cancel{21}_{1}\cdot\cancel{85}_{17}} = \frac{8}{17}$$

$$\Leftrightarrow \beta = \sin^{-1}\left(\frac{8}{17}\right) = 28°\,4'\,20{,}95''$$

$$\gamma = 180° - \alpha - \beta \simeq 53°\,7'\,48{,}37''$$

(3) $\frac{h_A\cdot a}{2} = [ABC] \Leftrightarrow h_A = \frac{2\cdot[ABC]}{a}$

$$\Leftrightarrow h_A = \frac{2\cdot 84}{21} = 8\text{ cm}$$

De même : $h_B = \frac{2\cdot[ABC]}{b} = \frac{{}^{1}\cancel{2}\cdot 84}{\cancel{10}_{5}} = \frac{84}{5}$ cm

$$h_C = \frac{2\cdot[ABC]}{c} = \frac{2\cdot 84}{17} = \frac{168}{17}\text{ cm}.$$