

*Durée : 60'**Calculatrice non autorisée*

Question 1

4 points

Compléter et démontrer : $\tan\left(\frac{\pi}{2} + \alpha\right) = \dots -\cot \alpha \dots$

Démonstration :

$$\tan\left(\frac{\pi}{2} + \alpha\right) = \frac{\sin\left(\frac{\pi}{2} + \alpha\right)}{\cos\left(\frac{\pi}{2} + \alpha\right)} = \frac{\cos \alpha}{-\sin \alpha} = -\cot \alpha$$

Question 2

15 (=5x3) points

Déterminer la valeur exacte en justifiant la réponse :

(1) $\cos\left(-\frac{47\pi}{6}\right)$

$$\begin{aligned}\cos\left(-\frac{47\pi}{6}\right) &= \cos\left(\frac{47\pi}{6}\right) = \cos\left(7\pi + \frac{5\pi}{6}\right) \\ &= -\cos\left(\frac{5\pi}{6}\right) = +\frac{\sqrt{3}}{2}\end{aligned}$$

(2) $\sin\left(\frac{15\pi}{2}\right)$

$$\begin{aligned}\sin\left(\frac{15\pi}{2}\right) &= \sin\left(7\pi + \frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) \\ &= -1\end{aligned}$$

(3) $\tan\left(-\frac{8\pi}{3}\right)$

$$\begin{aligned}\tan\left(-\frac{8\pi}{3}\right) &= -\tan\left(\frac{8\pi}{3}\right) = -\tan\left(2\pi + \frac{2\pi}{3}\right) \\ &= -\tan\left(\frac{2\pi}{3}\right) = +\sqrt{3}\end{aligned}$$

(4) $\sin\left(\frac{17\pi}{3}\right)$

$$\sin\left(\frac{17\pi}{3}\right) = \sin\left(5\pi + \frac{2\pi}{3}\right) = -\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

(5) $\cot\left(-\frac{55\pi}{4}\right)$

$$\begin{aligned}\cot\left(-\frac{55\pi}{4}\right) &= -\cot\left(\frac{55\pi}{4}\right) = -\cot\left(13\pi + \frac{3\pi}{4}\right) \\ &= -\cot\left(\frac{3\pi}{4}\right) = +1\end{aligned}$$

Question 3

4 points

- (1) Quelle est la mesure principale d'un angle de 279° ? ... $279 - 360 = -81^\circ$
- (2) Quelle est la plus petite mesure positive d'un angle de $-\frac{22\pi}{5}$ rad ? ... $\frac{8\pi}{5}$ rad

Question 4

12 (=4x3) points

Exprimer en fonction d'un angle de $[0, \frac{\pi}{4}]$ en justifiant la réponse :

(1) $\cos\left(\frac{37\pi}{8}\right)$

$$\begin{aligned}\cos\left(\frac{37\pi}{8}\right) &= \cos\left(4\pi + \frac{5\pi}{8}\right) = \cos\left(\frac{5\pi}{8}\right) \\ &= \cos\left(\pi - \frac{3\pi}{8}\right) = -\cos\left(\frac{3\pi}{8}\right) = -\sin\left(\frac{\pi}{2} - \frac{3\pi}{8}\right) \\ &= -\sin\left(\frac{\pi}{8}\right)\end{aligned}$$

(2) $\sin\left(-\frac{36\pi}{5}\right)$

$$\begin{aligned}\sin\left(-\frac{36\pi}{5}\right) &= -\sin\left(\frac{36\pi}{5}\right) = -\sin\left(7\pi + \frac{\pi}{5}\right) \\ &= \sin\frac{\pi}{5} \quad \left(\frac{\pi}{5} \in [0, \frac{\pi}{4}]\right)\end{aligned}$$

(3) $\tan\left(\frac{6\pi}{11}\right)$

$$\begin{aligned}\tan\left(\frac{6\pi}{11}\right) &= \tan\left(\frac{6\pi}{11} - \pi\right) = \tan\left(-\frac{5\pi}{11}\right) \\ &= -\tan\left(\frac{5\pi}{11}\right) = -\cot\left(\frac{\pi}{2} - \frac{5\pi}{11}\right) = -\cot\left(\frac{\pi}{22}\right)\end{aligned}$$

(4) $\cos\left(-\frac{49\pi}{9}\right)$

$$\begin{aligned}\cos\left(-\frac{49\pi}{9}\right) &= \cos\left(\frac{49\pi}{9}\right) = \cos\left(5\pi + \frac{4\pi}{9}\right) \\ &= -\cos\left(\frac{4\pi}{9}\right) = -\sin\left(\frac{\pi}{2} - \frac{4\pi}{9}\right) = -\sin\left(\frac{\pi}{18}\right)\end{aligned}$$

Question 5

14 (=8+6) points

Simplifier les expressions suivantes autant que possible :

(1)
$$\frac{\cos(\alpha - 3\pi) \cdot \sin\left(\frac{3\pi}{2} - \alpha\right)}{\cos\left(-\frac{5\pi}{2} + \alpha\right) \cdot \sin(4\pi - \alpha)}$$

$$\begin{aligned}&= \frac{-\cos(\alpha) \cdot (-\sin(\frac{\pi}{2} - \alpha))}{-\cos(\frac{\pi}{2} + \alpha) \cdot \sin(-\alpha)} \\ &= \frac{\cos \alpha \cdot \cos \alpha}{\sin \alpha \cdot (-\sin \alpha)} \\ &= -\frac{\cos^2 \alpha}{\sin^2 \alpha} = -\cot^2(\alpha)\end{aligned}$$

$$(2) \quad \cos\left(\alpha - \frac{3\pi}{4}\right) + \cos\left(\alpha - \frac{\pi}{4}\right) + \cos\left(\alpha + \frac{\pi}{4}\right) + \cos\left(\alpha + \frac{3\pi}{4}\right)$$

$$= -\cancel{\cos\left(\alpha + \frac{\pi}{4}\right)} + \cancel{\cos\left(\alpha - \frac{\pi}{4}\right)} + \cancel{\cos\left(\alpha + \frac{\pi}{4}\right)} - \cancel{\cos\left(\alpha - \frac{\pi}{4}\right)}$$

$$= 0$$

Question 6

11 (=7+4) points

(1) Résoudre l'équation $\sin x = -\frac{\sqrt{3}}{2}$ a) dans \mathbb{R} et b) dans $[-4\pi, 4\pi]$

$$a) \sin x = -\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow x = -\frac{\pi}{3} + k \cdot 2\pi \text{ ou } x = -\frac{2\pi}{3} + k \cdot 2\pi, k \in \mathbb{Z}$$

$$S = \left\{ -\frac{\pi}{3} + k \cdot 2\pi, -\frac{2\pi}{3} + k \cdot 2\pi \mid k \in \mathbb{Z} \right\}$$

$$b) S' = \left\{ -\frac{7\pi}{3}, -\frac{8\pi}{3}, -\frac{\pi}{3}, -\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3} \right\}$$

(2) Résoudre l'équation $\cos x = -1$ a) dans \mathbb{R} et b) dans $[-3\pi, 3\pi]$

$$a) \cos x = -1 \Leftrightarrow x = \pi + k \cdot 2\pi, k \in \mathbb{Z}$$

$$S = \{ \pi + k \cdot 2\pi, k \in \mathbb{Z} \}$$

$$b) S' = \{ -3\pi, -\pi, \pi, 3\pi \}$$

G. Lorang