3eC2 — Corrigé du devoir II, 3 — 15.03.10

Question 1

$$2\sin\left(\frac{x}{2}\right)=\sqrt{3}$$

$$\Leftrightarrow \sin\left(\frac{x}{2}\right)=\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \begin{cases} \dfrac{x}{2}=\dfrac{\pi}{3}+k\cdot 2\pi \\ \text{ou} \\ \dfrac{x}{2}=\dfrac{2\pi}{3}+k\cdot 2\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x=\dfrac{2\pi}{3}+k\cdot 4\pi \\ \text{ou} \\ x=\dfrac{4\pi}{3}+k\cdot 4\pi, \quad k\in\mathbb{Z} \end{cases}$$

$$S=\left\{\frac{2\pi}{3}+k\cdot 4\pi\,;\ \frac{4\pi}{3}+k\cdot 4\pi \,/\, k\in\mathbb{Z}\right\}$$

Solutions dans $[-4\pi, 4\pi]$:

$$\frac{2\pi}{3}\,;\ \frac{4\pi}{3}\,;\ -\frac{10\pi}{3}\,;\ -\frac{8\pi}{3}$$

Question 2

(1) $$\cos\left(3x+\frac{\pi}{2}\right)=0{,}6$$

$$\Leftrightarrow \begin{cases} 3x+\dfrac{\pi}{2}=0{,}9273+k\cdot 2\pi \\ \text{ou} \\ 3x+\dfrac{\pi}{2}=-0{,}9273+k\cdot 2\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} 3x=-0{,}6435+k\cdot 2\pi \\ \text{ou} \\ 3x=-2{,}4981+k\cdot 2\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x=-0{,}2145+k\cdot\dfrac{2\pi}{3} \\ x=-0{,}8327+k\cdot\dfrac{2\pi}{3}, \quad k\in\mathbb{Z} \end{cases}$$

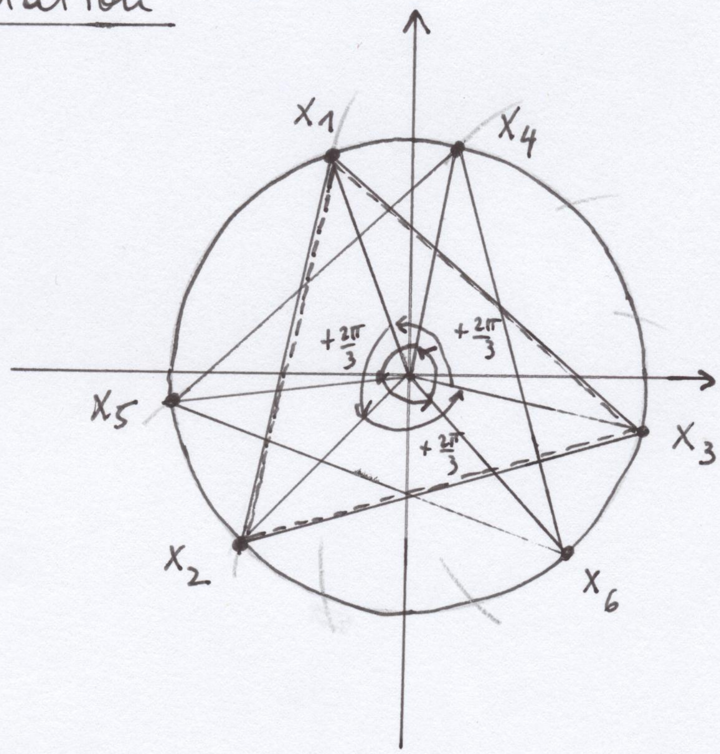
$$S=\left\{-0{,}2145+k\cdot\frac{2\pi}{3}\,;\ -0{,}8327+k\cdot\frac{2\pi}{3} \,/\, k\in\mathbb{Z}\right\}$$

(2) Solutions dans $[0, 2\pi]$:

$$\begin{cases} \underbrace{1{,}8799}_{x_1}\,;\ \underbrace{3{,}9743}_{x_2}\,;\ \underbrace{6{,}0687}_{x_3} \\ \underbrace{1{,}2617}_{x_4}\,;\ \underbrace{3{,}3561}_{x_5}\,;\ \underbrace{5{,}4505}_{x_6} \end{cases}$$

Rem : $-0{,}2145$ rad $\simeq -12{,}3°$
$-0{,}8327$ rad $\simeq -47{,}7°$

Représentation



Question 3

$$\sqrt{3}\left(\tan^2 x - 1\right) = 2\tan x \qquad \underline{C.E}: \; x \neq \frac{\pi}{2} + k\pi, \; k \in \mathbb{Z}$$

Posons : $y = \tan x$

Alors l'éq. devient :

$$\sqrt{3}\,y^2 - \sqrt{3} = 2y$$

$$\Leftrightarrow \sqrt{3}\,y^2 - 2y - \sqrt{3} = 0$$

$$\Delta = 4 - 4\cdot\sqrt{3}(-\sqrt{3}) = 16$$

$$y_1 = \frac{2-4}{2\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

$$y_2 = \frac{2+4}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

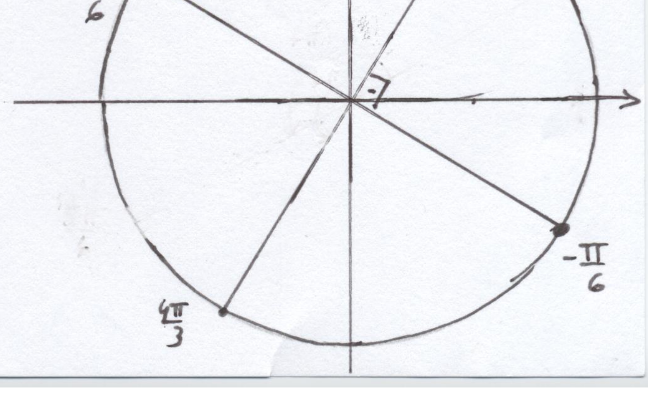
Revenons à x :

$$\tan x = -\frac{1}{\sqrt{3}} \Leftrightarrow x = -\frac{\pi}{6} + k\pi$$

ou $\tan x = \sqrt{3} \Leftrightarrow x = \frac{\pi}{3} + k\pi, k \in \mathbb{Z}$

$$S = \left\{-\frac{\pi}{6} + k\pi \; ; \; \frac{\pi}{3} + k\pi \,/\, k \in \mathbb{Z}\right\}$$

$$= \left\{-\frac{\pi}{6} + k\cdot\frac{\pi}{2} \,/\, k \in \mathbb{Z}\right\}$$



Question 4.

(1) $D = \mathbb{R}$

$$\cos^4 a - \sin^4 a$$
$$= (\cos^2 a - \sin^2 a)(\underbrace{\cos^2 a + \sin^2 a}_{1})$$
$$= \cos^2 a - \sin^2 a$$
$$= \cos^2 a - (1 - \cos^2 a)$$
$$= \cos^2 a - 1 + \cos^2 a$$
$$= 2\cos^2 a - 1$$

(2) C.E : $\begin{cases} x \neq \frac{\pi}{2} + k\cdot\pi \ , \ k \in \mathbb{Z} & (1) \\ \tan^2 x \neq 1 & (2) \\ \sin^2 x - \cos^2 x \neq 0 & (3) \end{cases}$

(2) $\Leftrightarrow \tan x \neq \pm 1$

$\Leftrightarrow x \neq \frac{\pi}{4} + k\cdot\pi$ et $x \neq -\frac{\pi}{4} + k\cdot\pi$

$\Leftrightarrow x \neq \frac{\pi}{4} + k\cdot\frac{\pi}{2}$

(3) $\sin^2 x - \cos^2 x \neq 0$

$\Leftrightarrow 1 - \cos^2 x - \cos^2 x \neq 0$

$\Leftrightarrow \cos^2 x \neq \frac{1}{2}$

$\Leftrightarrow \cos x \neq \frac{1}{\sqrt{2}}$ et $\cos x \neq -\frac{1}{\sqrt{2}}$

$\Leftrightarrow \begin{cases} x \neq \frac{\pi}{4} + k\cdot 2\pi \\ x \neq -\frac{\pi}{4} + k\cdot 2\pi \\ x \neq \frac{3\pi}{4} + k\cdot 2\pi \\ x \neq -\frac{3\pi}{4} + k\cdot 2\pi \end{cases} \Leftrightarrow x \neq \frac{\pi}{4} + k\cdot\frac{\pi}{2}$

$$D = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\cdot\pi \ ; \ \frac{\pi}{4} + k\cdot\frac{\pi}{2} \ / \ k \in \mathbb{Z} \right\}$$

$(\forall x \in D)$ $$\frac{\tan x}{\tan^2 x - 1} = \frac{\frac{\sin x}{\cos x}}{\frac{\sin^2 x}{\cos^2 x} - 1}$$

$$= \frac{\frac{\sin x}{\cos x}}{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}}$$

$$= \frac{\sin x}{\cancel{\cos x}} \cdot \frac{\cos^{\cancel{2}} x}{\sin^2 x - \cos^2 x} = \frac{\sin x \cos x}{\sin^2 x - \cos^2 x}$$