

Question 1

Voir cours

Question 2

- (1) Soit $P(x) = x^3 - 4x^2 - 8x + 8$
 $\text{Div}(8) = \{\pm 1; \pm 2; \pm 4; \pm 8\}$
 $P(1) = 1 - 4 - 8 + 8 = -3 \neq 0$
 $P(-1) = -1 - 4 + 8 + 8 = 11 \neq 0$
 $P(2) = 8 - 16 - 16 + 8 = -16 \neq 0$
 $P(-2) = -8 - 16 + 16 + 8 = 0$
 Donc $P(x)$ est divisible par $x+2$

$$\begin{array}{c|c|c|c|c} & 1 & -4 & -8 & 8 \\ -2 & & -2 & 12 & -8 \\ \hline & 1 & -6 & 4 & 0 \end{array}$$

$$P(x) = (x+2) \cdot (x^2 - 6x + 4)$$

$$\text{Posons } Q(x) = x^2 - 6x + 4$$

$$\Delta = 36 - 4 \cdot 1 \cdot 4 = 20 > 0$$

$$x_1 = \frac{6 - \sqrt{20}}{2} = \frac{6 - 2\sqrt{5}}{2} = 3 - \sqrt{5}$$

$$x_2 = 3 + \sqrt{5}$$

$$\text{Donc } P(x) = 0 \Leftrightarrow x = -2 \text{ ou } x = 3 \pm \sqrt{5}$$

$$S = \{-2; 3 \pm \sqrt{5}\}$$

$$(2) \quad \frac{1}{x^2 - x - 2} - \frac{x+1}{x^2 - 4} = \frac{1}{x^2 + 3x + 2}$$

$$\Leftrightarrow \frac{1}{(x+1)(x-2)} - \frac{x+1}{(x-2)(x+2)} = \frac{1}{(x+1)(x+2)}$$

$$\text{C.E: } x \neq -1, x \neq 2, x \neq -2$$

$$\mathcal{D} = \mathbb{R} \setminus \{-1; 2; -2\}$$

$$\Leftrightarrow \frac{x+2}{(x+1)(x-2)(x+2)} - \frac{(x+1)^2}{(x+1)(x-2)(x+2)} = \frac{x-2}{(x+1)(x-2)(x+2)} \quad | \cdot (x+1)(x-2)(x+2)$$

$$\Leftrightarrow \cancel{x+2} - (x^2 + 2x + 1) = \cancel{x-2}$$

$$\Leftrightarrow 2 - x^2 - 2x - 1 = -2$$

$$\Leftrightarrow -x^2 - 2x + 3 = 0 \quad | \cdot (-1)$$

$$\Leftrightarrow x^2 + 2x - 3 = 0$$

$$\Leftrightarrow (x-1)(x+3) = 0$$

$$\Leftrightarrow \underbrace{x=1}_{\in \mathcal{D}} \text{ ou } \underbrace{x=-3}_{\in \mathcal{D}}$$

$$S = \{1; -3\}$$

Question 3

$$(1) \quad 3x - 4x^3 \geq 0$$

$$\Leftrightarrow x(3 - 4x^2) \geq 0$$

V.C. : • $x = 0$
 • $3 - 4x^2 = 0 \Leftrightarrow x^2 = \frac{3}{4} \Leftrightarrow x = \pm \frac{\sqrt{3}}{2}$

x	$-\infty$	$-\frac{\sqrt{3}}{2}$	0	$\frac{\sqrt{3}}{2}$	$+\infty$
x		-	0	+	+
$3 - 4x^2$	-	0	+	0	-
$x(3 - 4x^2)$	+	0	-	0	-

$$S = \left] -\infty; -\frac{\sqrt{3}}{2} \right] \cup \left[0; \frac{\sqrt{3}}{2} \right]$$

$$(2) \quad \underbrace{(2 - 3x)(2x^2 + 5)(6x^2 - x - 1)}_{P(x)} < 0$$

V.C. : • $2 - 3x = 0 \Leftrightarrow x = \frac{2}{3}$
 • $2x^2 + 5 = 0$, impossible
 • $6x^2 - x - 1 = 0$

$$\Delta = 1 + 24 = 25$$

$$x_1 = \frac{1 - 5}{12} = -\frac{4}{12} = -\frac{1}{3}$$

$$x_2 = \frac{1 + 5}{12} = \frac{6}{12} = \frac{1}{2}$$

x	$-\infty$	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$+\infty$
$2 - 3x$	+	+	+	0	-
$2x^2 + 5$	+	+	+	+	+
$6x^2 - x - 1$	+	0	-	0	+
$P(x)$	+	0	-	0	-

$$S = \left] -\frac{1}{3}; \frac{1}{2} \right[\cup \left] \frac{2}{3}; +\infty \right[$$

Question 4

Voir cours