Devoir II,1 corrigé

Dienstag, 28. Januar 2020 08:58

Question 2

$$(\cos^2 x + \sin^2 x)^2 + (\cos^2 x - \sin^2 x)^2 = 2 - 4\cos^2 x \sin^2 x$$

($\cos^2 x + \sin^2 x = 1$)

$$\Leftrightarrow 1^2 + \cos^4 x - 2\cos^2 x \sin^2 x + \sin^4 x = 2 - 4\cos^2 x \sin^2 x$$

$$\Leftrightarrow \cos^4 x - 2\cos^2 x \sin^2 x + 4\cos^2 x \sin^2 x + \sin^4 x = 2 - 1$$

$$\Leftrightarrow \cos^4 x + 2\cos^2 x \sin^2 x + \sin^4 x = 1$$

$$\Leftrightarrow (\cos^2 x + \sin^2 x)^2 = 1$$

$$\Leftrightarrow 1^2 = 1 \quad \text{VRAI !}$$

Question 3

(1) $\sin\left(-\frac{58\pi}{9}\right) = -\sin\left(\frac{58\pi}{9}\right) = -\sin\left(6\pi + \frac{4\pi}{9}\right)$

$= -\sin\left(\frac{4\pi}{9}\right) = -\cos\left(\frac{\pi}{2} - \frac{4\pi}{9}\right)$

$= -\cos\frac{\pi}{18}$

(2) a) $\cos(25\pi - x) = \cos(\pi - x) = -\cos x$

b) $\sin\left(x + \frac{7\pi}{2}\right) = \sin\left(x - \frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2} - x\right) = -\cos x$

c) $\tan\left(\frac{3\pi}{2} - x\right) = \tan\left(\frac{\pi}{2} - x\right) = \cot x$

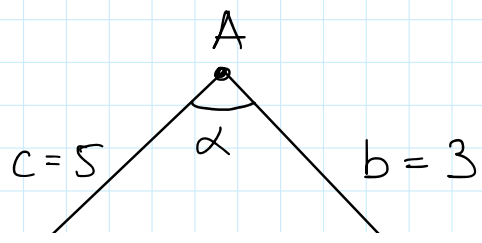
d) $\sin(48\pi - x) = \sin(-x) = -\sin x$

e) $\cos\left(\frac{13\pi}{2} + x\right) = \cos\left(\frac{\pi}{2} + x\right) = -\sin x$

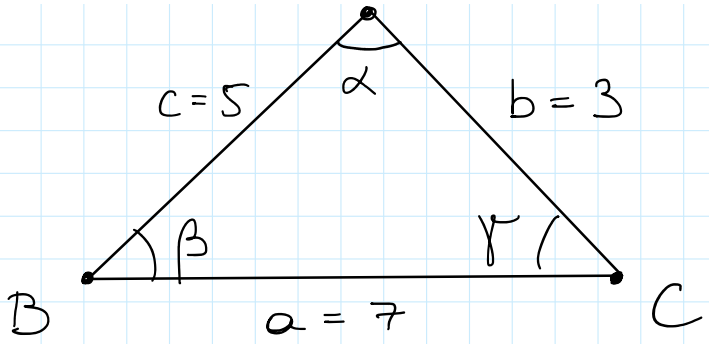
f) $\cot(3\pi - x) = \cot(-x) = -\cot x$

Question 4

(1)



c)



a) Al-Kashi :

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$\Leftrightarrow 2bc\cos\alpha = b^2 + c^2 - a^2$$

$$\Leftrightarrow \cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$\Leftrightarrow \cos\alpha = \frac{25+9-49}{2\cdot 5\cdot 3} = -\frac{15}{30} = -\frac{1}{2}$$

$$\Leftrightarrow \alpha = 120° \text{ (obtus !)}$$

b) Donc β et γ sont des angles aigus !

$$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha} \quad \text{(relation aux sinus)}$$

$$\Leftrightarrow \frac{3}{\sin\beta} = \frac{7}{\sqrt{3}/2}$$

$$\Leftrightarrow 7\sin\beta = \frac{3\sqrt{3}}{2} \quad |:7$$

$$\Leftrightarrow \sin\beta = \frac{3\sqrt{3}}{14}$$

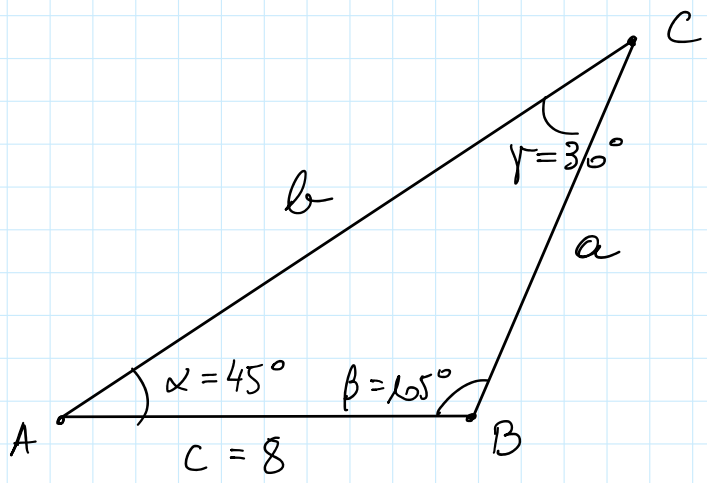
$$\Leftrightarrow \beta \approx 21° 47' 12{,}44'' \quad (\beta \text{ aigu !})$$

c) $\gamma = 180° - \alpha - \beta \approx 38° 12' 47{,}56''$

d) $[ABC] = \frac{b\cdot c\cdot\sin\alpha}{2} = \frac{3\cdot 5\cdot\sin 120°}{2}$

$$= \frac{15\cdot\sqrt{3}}{2\cdot 2} = \frac{15\sqrt{3}}{4} \text{ u.a.}$$

(2)



a) $\gamma = 180° - 45° - 105° = 30°$

b) Relation aux sinus :

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\Leftrightarrow \frac{a}{\sin 45°} = \frac{8}{\sin 30°}$$

$$\Leftrightarrow \frac{a}{\sqrt{2}/2} = \frac{8}{1/2}$$

$$\Leftrightarrow \frac{1}{2}a = \frac{\sqrt{2}}{2} \cdot 8$$

$$\Leftrightarrow a = 8\sqrt{2}$$

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\Leftrightarrow \frac{b}{\sin 105°} = \frac{8}{\sin 30°}$$

$$\Leftrightarrow b = \frac{8 \cdot \sin 105°}{\sin 30°}$$

$$\Leftrightarrow b = 4\sqrt{6} + 4\sqrt{2}$$

$$\approx 15{,}4548$$

$$[ABC] = \frac{a \cdot c \cdot \sin\beta}{2}$$

$$= \frac{8\sqrt{2} \cdot 8 \cdot \sin 105°}{2}$$

$$= 32\sqrt{2} \cdot \sin 105°$$

$$= 16 + 16\sqrt{3}$$

$$\approx 43{,}7128 \text{ u.a.}$$

Question 5

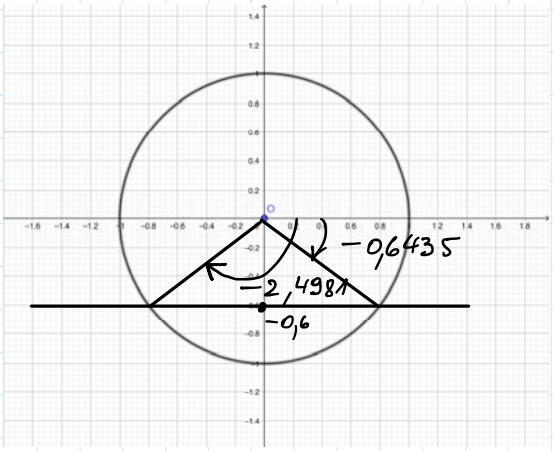
(1) $\sin x = -\frac{3}{5}$

$\Leftrightarrow x \approx -0{,}6435 + k \cdot 2\pi$

ou $x \approx 3{,}7851 + k \cdot 2\pi$, $k \in \mathbb{Z}$

$S = \{-0{,}6435 + k \cdot 2\pi ; 3{,}7851 + k \cdot 2\pi \mid k \in \mathbb{Z}\}$

2)



3) $-0{,}6435 + 4\pi \approx 11{,}9229$

$-2{,}4981 + 4\pi \approx 10{,}0683$