3CMB1/B2 Corrigé du devoir de mathématiques I,2 7.11.03

Exercice 1

(1) Voir manuel.

(2) Voir cahier.

Exercice 2

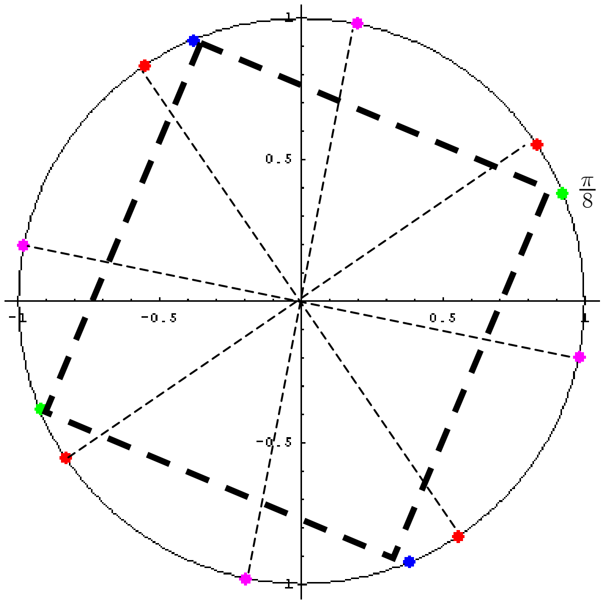
$$\sin^2(3x) = \cos^2\left(x - \tfrac{\pi}{4}\right)$$

$$\Leftrightarrow \begin{cases} \sin(3x) = \cos\left(x - \frac{\pi}{4}\right) \text{ ou} \\ -\sin(3x) = \cos\left(x - \frac{\pi}{4}\right) \end{cases}$$

$$\Leftrightarrow \begin{cases} \cos\left(\frac{\pi}{2} - 3x\right) = \cos\left(x - \frac{\pi}{4}\right) \text{ ou} \\ \cos\left(\frac{\pi}{2} + 3x\right) = \cos\left(x - \frac{\pi}{4}\right) \end{cases}$$

$$\Leftrightarrow \begin{cases} \frac{\pi}{2} - 3x = x - \frac{\pi}{4} + k\cdot 2\pi \text{ ou} \\ \frac{\pi}{2} - 3x = -x + \frac{\pi}{4} + k\cdot 2\pi \text{ ou} \\ \frac{\pi}{2} + 3x = x - \frac{\pi}{4} + k\cdot 2\pi \text{ ou} \\ \frac{\pi}{2} + 3x = -x + \frac{\pi}{4} + k\cdot 2\pi \end{cases}, \ k \in \mathbb{Z}$$

$$\Leftrightarrow \begin{cases} x = \frac{3\pi}{16} - k\cdot\frac{\pi}{2} \text{ ou} \\ x = \frac{\pi}{8} - k\cdot\pi \text{ ou} \\ x = -\frac{3\pi}{8} + k\cdot\pi \text{ ou} \\ x = -\frac{\pi}{16} + k\cdot\frac{\pi}{2} \end{cases}$$



$$S = \left\{\tfrac{\pi}{8} + k\cdot\tfrac{\pi}{2}, -\tfrac{\pi}{16} + k\cdot\tfrac{\pi}{4} \,/\, k \in \mathbb{Z}\right\}$$

Exercice 3

Posons $a = 13$ cm, $b = 14$ cm et $c = 15$ cm . Appliquons la relation au cosinus :

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} = \frac{14^2 + 15^2 - 13^2}{2\cdot 14\cdot 15} = \frac{252}{2\cdot 14\cdot 15} = \frac{3}{5}$$

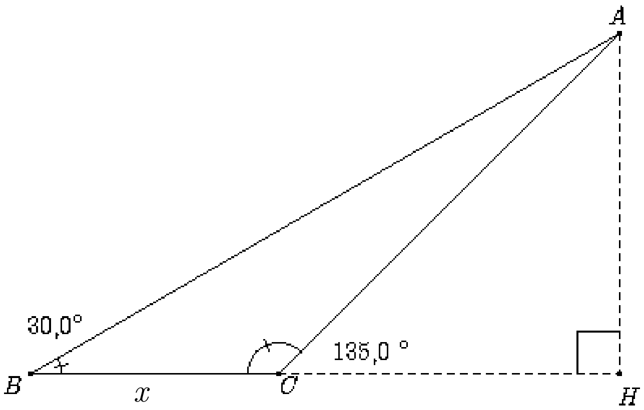
On déduit de la relation fondamentale de la trigonométrie que :

$$\sin\alpha = \sqrt{1 - \cos^2\alpha} = \sqrt{1 - \tfrac{9}{25}} = \sqrt{\tfrac{16}{25}} = \tfrac{4}{5}$$

Par conséquent, l'aire du triangle vaut :

$$S = \tfrac{1}{2}bc\sin\alpha = \tfrac{1}{2}\cdot 14\cdot 15\cdot\tfrac{4}{5} = 28\cdot 3 = 84 \text{ cm}^2$$

Exercice 4



(1) On remarque que $\hat{A} = 180° - \hat{B} - \hat{C} = 15°$. On peut alors appliquer la relation aux sinus :

$$\frac{\overline{AB}}{\sin 135°} = \frac{\overline{BC}}{\sin 15°} = \frac{\overline{CA}}{\sin 30°}$$

$$\Leftrightarrow \frac{\overline{AB}}{\frac{\sqrt{2}}{2}} = \frac{x}{\frac{\sqrt{6}-\sqrt{2}}{4}} = \frac{\overline{CA}}{\frac{1}{2}}$$

$$\Leftrightarrow \begin{cases} \overline{AB} = \dfrac{4x}{\sqrt{6}-\sqrt{2}} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{4\sqrt{2}\left(\sqrt{6}+\sqrt{2}\right)x}{8} = \dfrac{2\sqrt{3}+2}{2}x = \left(\sqrt{3}+1\right)x \\ \overline{CA} = \dfrac{4x}{\sqrt{6}-\sqrt{2}} \cdot \dfrac{1}{2} = \dfrac{4\left(\sqrt{6}+\sqrt{2}\right)x}{8} = \dfrac{\sqrt{6}+\sqrt{2}}{2}x \end{cases}$$

(2) $S = \frac{1}{2}\overline{BA} \cdot \overline{BC} \cdot \sin \hat{B} = \frac{1}{2}\left(\sqrt{3}+1\right)x^2 \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{4}x^2$

(3) Soit H le pied de la hauteur issue de A. On sait que l'aire du triangle est donnée par la formule : $S = \dfrac{\overline{BC} \cdot \overline{AH}}{2}$. Donc :

$$\overline{AH} = \frac{2S}{\overline{BC}} = \frac{\frac{\sqrt{3}+1}{2}x^2}{x} = \frac{\sqrt{3}+1}{2}x\,.$$

G. Lorang