Question 1

(1) $EJ \,//\, BI \Rightarrow \dfrac{\overline{AJ}}{\overline{AI}} = \dfrac{\overline{AE}}{\overline{AB}} \Leftrightarrow \dfrac{\overline{AJ}}{6} = \dfrac{5}{8} \Leftrightarrow \overline{AJ} = \dfrac{15}{4}$

(2) $EF \,//\, BC \Rightarrow \dfrac{\overline{AC}}{\overline{FC}} = \dfrac{\overline{AB}}{\overline{EB}} \Leftrightarrow \dfrac{\overline{AC}}{4} = \dfrac{8}{3} \Leftrightarrow \overline{AC} = \dfrac{32}{3}$

(3) $EF \,//\, BC \Rightarrow \dfrac{\overline{JF}}{\overline{IC}} = \dfrac{\overline{AJ}}{\overline{AI}} = \dfrac{\overline{AE}}{\overline{AB}} \Leftrightarrow \dfrac{\overline{JF}}{8} = \dfrac{5}{8} \Leftrightarrow \overline{JF} = 5$

(4) $\overline{EJ} = \overline{EF} - \overline{JF} = 7 - 5 = 2$, donc :

$\dfrac{\overline{JF}}{\overline{IC}} = \dfrac{\overline{BI}}{\overline{EJ}} = \dfrac{\overline{AB}}{\overline{AE}} \Leftrightarrow \dfrac{\overline{BI}}{2} = \dfrac{8}{5} \Leftrightarrow \overline{BI} = \dfrac{16}{5}$.

Question 2

(1) D'après la relation fondamentale de la trigonométrie :

$$\begin{aligned} &\sin^2 \hat{A} + \cos^2 \hat{A} = 1 \\ &\Leftrightarrow \sin^2 \hat{A} = 1 - \cos^2 \hat{A} \\ &\Leftrightarrow \sin^2 \hat{A} = 1 - \tfrac{144}{169} = \tfrac{25}{169} \,/\sqrt{\ } \\ &\Leftrightarrow \sin \hat{A} = \tfrac{5}{13} \end{aligned}$$

Donc : $\tan \hat{A} = \dfrac{\sin \hat{A}}{\cos \hat{A}} = \dfrac{5}{12}$.

(2) On sait que :

$$\begin{aligned} &\cos^2 \hat{B} = \frac{1}{1+\tan^2 \hat{B}} && \sin^2 \hat{B} = \frac{\tan^2 \hat{B}}{1+\tan^2 \hat{B}} \\ &\Leftrightarrow \cos^2 \hat{B} = \frac{1}{1+8} = \frac{1}{9} \quad \text{et} && \Leftrightarrow \sin^2 \hat{B} = \frac{8}{1+8} = \frac{8}{9} \\ &\Leftrightarrow \cos \hat{B} = \frac{1}{3} && \Leftrightarrow \sin \hat{B} = \frac{2\sqrt{2}}{3} \end{aligned}$$

(3) $\hat{A} = \cos^{-1}\left(\tfrac{12}{13}\right) \cong 22°\ 37'\ 11{,}5''$ et $\hat{B} = \cos^{-1}\left(\tfrac{1}{3}\right) \cong 70°\,31'\ 43{,}6''$.

Question 3

$\cos \hat{P}_1 = \dfrac{1050}{1850} \Leftrightarrow \hat{P}_1 = \cos^{-1}\left(\dfrac{1050}{1850}\right) \cong 55{,}42°$

$\cos\left(\hat{P}_1 + \hat{P}_2\right) = \dfrac{1050}{2460} \Leftrightarrow \hat{P} = \hat{P}_1 + \hat{P}_2 = \cos^{-1}\left(\dfrac{1050}{2460}\right) \cong 64{,}73°$

L'angle sous lequel le pilote voit le lac est donc :

$\hat{P}_2 = \hat{P} - \hat{P}_1 \simeq 64{,}73° - 55{,}42° \cong 9{,}31°$.

$$\tan \hat{P}_1 = \frac{\overline{QA}}{1050} \Leftrightarrow \overline{QA} = 1050 \cdot \tan \hat{P}_1$$

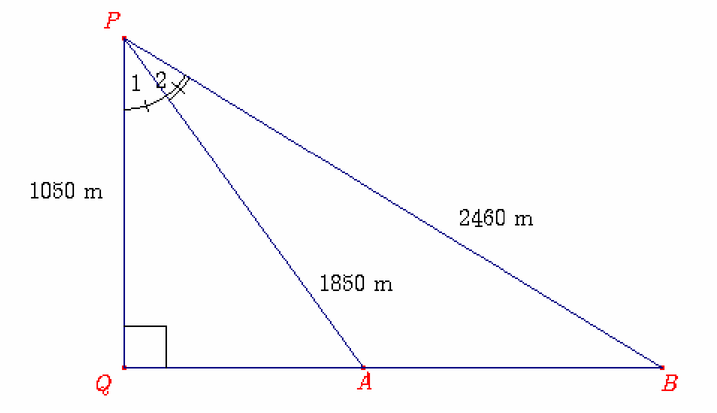
$$\tan \hat{P} = \frac{\overline{QB}}{1050} \Leftrightarrow \overline{QB} = 1050 \cdot \tan \hat{P}$$

La longueur du lac vaut donc :

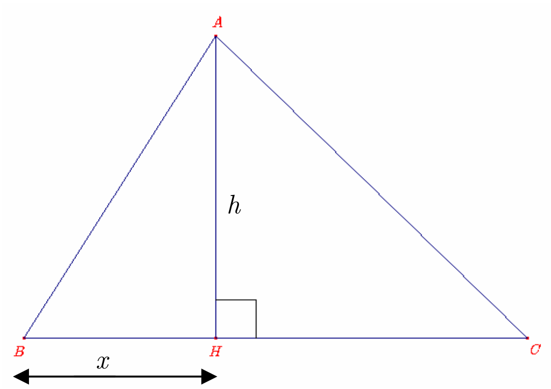
$$\overline{AB} = \overline{QB} - \overline{QA}$$

$$= 1050 \cdot \tan \hat{P} - 1050 \cdot \tan \hat{P}_1$$

$$\cong 701{,}5 \text{ m.}$$



Question 4



(1) $\overline{AB}^2 = h^2 + x^2 \Leftrightarrow 100 = h^2 + x^2$

$\Leftrightarrow h^2 = 100 - x^2 \qquad$ (a)

(2) $\overline{AC}^2 = h^2 + \overline{HC}^2 \Leftrightarrow 144 = h^2 + (16 - x)^2$

$\Leftrightarrow h^2 = 144 - (16 - x)^2 \qquad$ (b)

(3) Comparant les relations (a) et (b), on a :

$100 - x^2 = 144 - (16 - x)^2$

$\Leftrightarrow 100 - x^2 = 144 - (256 - 32x + x^2)$

$\Leftrightarrow 100 \cancel{-x^2} = 144 - 256 + 32x \cancel{-x^2}$

$\Leftrightarrow 100 = -112 + 32x$

$\Leftrightarrow 32x = 212$

$\Leftrightarrow x = \frac{212}{32} = \frac{53}{8}$

Substituant ce résultat dans (a), il vient :

$$h^2 = 100 - \left(\frac{53}{8}\right)^2 = \frac{3591}{64}$$

$$\Leftrightarrow h = \frac{3\sqrt{399}}{8}$$

(3) $\tan \hat{B} = \frac{h}{x} \Leftrightarrow \hat{B} = \tan^{-1}\left(\frac{3\sqrt{399}}{53}\right) \simeq 48{,}51°\,.$

$\tan \hat{C} = \frac{h}{16 - x} \Leftrightarrow \hat{C} = \tan^{-1}\left(\frac{\frac{3\sqrt{399}}{8}}{16 - \frac{53}{8}}\right) = \tan^{-1}\left(\frac{3\sqrt{399}}{75}\right) \simeq 38{,}62°\,.$

$\hat{A} = 180° - \hat{B} - \hat{C} \cong 92{,}51°\,.$

(4) L'aire du triangle ABC vaut : $\frac{\overline{BC} \cdot h}{2} = \frac{16 \cdot \frac{3\sqrt{399}}{8}}{2} = 3\sqrt{399} \cong 59{,}92$ u.a.

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