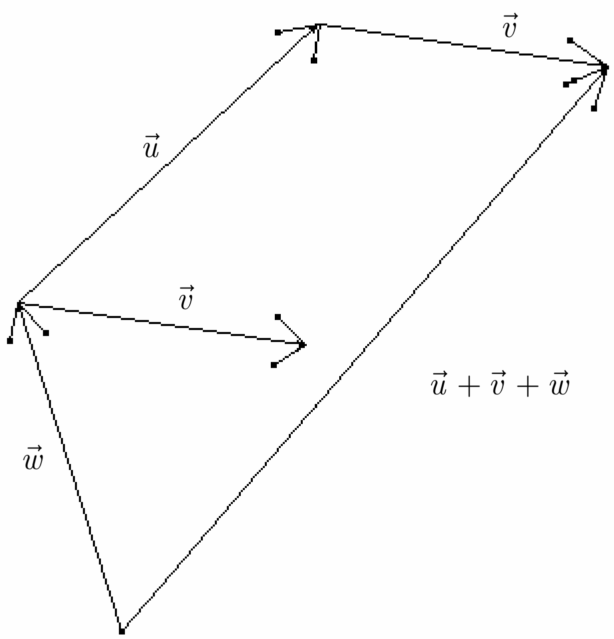
Nom : ………………… Prénom : …………………

4M6 Corrigé du devoir de mathématiques III,1 17.05.04

Exercice 1

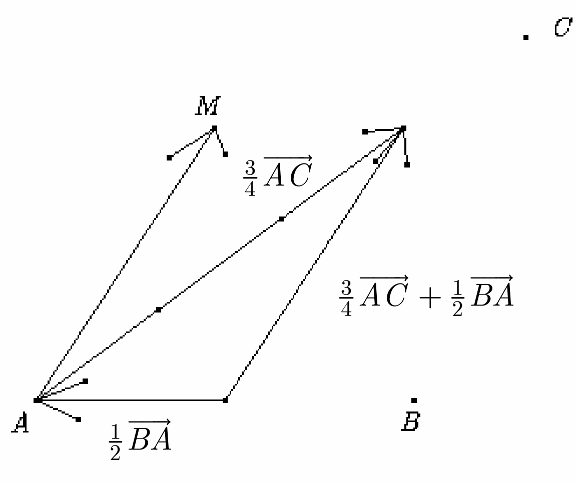
(1)



(2) a)

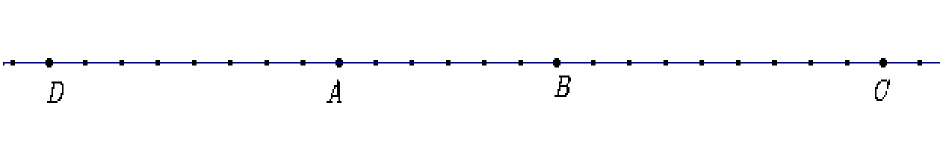
$$\overrightarrow{AM} + 2\cdot\overrightarrow{AB} = 3\cdot\overrightarrow{MC}$$
$$\Leftrightarrow \overrightarrow{AM} + 2\cdot\overrightarrow{AB} = 3\cdot\left(\overrightarrow{MA} + \overrightarrow{AC}\right)$$
$$\Leftrightarrow \overrightarrow{AM} - 3\cdot\overrightarrow{MA} = 3\cdot\overrightarrow{AC} - 2\cdot\overrightarrow{AB}$$
$$\Leftrightarrow 4\cdot\overrightarrow{AM} = 3\cdot\overrightarrow{AC} + 2\cdot\overrightarrow{BA}$$
$$\Leftrightarrow \overrightarrow{AM} = \tfrac{3}{4}\cdot\overrightarrow{AC} + \tfrac{1}{2}\cdot\overrightarrow{BA}$$

b)



Exercice 2

(1) et (2) :



(3)

$$\begin{aligned}\overrightarrow{CD} &= \overrightarrow{CB}+\overrightarrow{BA}+\overrightarrow{AD}\\ &= -\tfrac{5}{2}\overrightarrow{AB}-\overrightarrow{AB}-\tfrac{4}{3}\overrightarrow{AB}\\ &= \left(-\tfrac{5}{2}-1-\tfrac{4}{3}\right)\overrightarrow{AB}\\ &= -\tfrac{29}{6}\overrightarrow{AB}\end{aligned}$$

(4)

$$\begin{aligned}\overline{CD} &= \tfrac{29}{6}\cdot 1{,}5\\ &= \tfrac{29}{6}\cdot\tfrac{3}{2}\\ &= \tfrac{29}{4} = 7{,}25\text{ cm}\end{aligned}$$

Exercice 3

(1) $A(0{,}0)$, $B(1{,}0)$, $C(2{,}0)$, $D(3{,}0)$, $E(0{,}1)$, $F(1{,}1)$, $G(2{,}1)$, $H(0{,}2)$, $I(1{,}2)$, $J(0{,}3)$.

(2) $\overrightarrow{FJ}\begin{pmatrix}0-1\\3-1\end{pmatrix}=\overrightarrow{FJ}\begin{pmatrix}-1\\2\end{pmatrix}$, $\overrightarrow{FD}\begin{pmatrix}3-1\\0-1\end{pmatrix}=\overrightarrow{FA}\begin{pmatrix}2\\-1\end{pmatrix}$.

$\overrightarrow{FA}\begin{pmatrix}0-1\\0-1\end{pmatrix}=\overrightarrow{FA}\begin{pmatrix}-1\\-1\end{pmatrix}$,

(3) $\overrightarrow{FJ}+\overrightarrow{FA}+\overrightarrow{FD}\begin{pmatrix}-1-1+2\\2-1-1\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$. Donc $\overrightarrow{FJ}+\overrightarrow{FA}+\overrightarrow{FD}=\vec{0}$.

(F est le centre de gravité du triangle JAD).

Exercice 4

(1) $\det(\vec{u},\vec{v})=\begin{vmatrix}\frac{1}{3} & 3\\ -2 & -\frac{1}{5}\end{vmatrix}=-\frac{1}{15}+6=\frac{89}{15}\neq 0$, donc $\vec{u}\ \nparallel\ \vec{v}$.

(2) $\vec{w}=2\vec{u}-\frac{1}{2}\vec{v}\Leftrightarrow\begin{cases}a=2\cdot\frac{1}{3}-\frac{1}{2}\cdot 3=-\frac{5}{6}\\ b=2\cdot(-2)-\frac{1}{2}\left(-\frac{1}{5}\right)=-4+\frac{1}{10}=-\frac{39}{10}\end{cases}$, donc $\vec{w}\begin{pmatrix}-\frac{5}{6}\\-\frac{39}{10}\end{pmatrix}$.

(3) $\vec{z}\parallel(\vec{u}-\vec{v})\Leftrightarrow\begin{vmatrix}4 & \frac{1}{3}-3\\ y & -2+\frac{1}{5}\end{vmatrix}=0\Leftrightarrow 4\cdot\left(-\frac{9}{5}\right)+\frac{8}{3}y=0\Leftrightarrow\frac{8}{3}y-\frac{36}{5}=0\Leftrightarrow y=\frac{36}{5}\cdot\frac{3}{8}=\frac{27}{10}$

G. Lorang