

Exercice 1

- (1) $(\forall a, b \in \mathbb{R}_+) \sqrt{ab} = \sqrt{a}\sqrt{b}$. Pour la démonstration, voir cours.
- (2) a) $(\forall a \in \mathbb{R}) \sqrt{a^2} = |a|$
 b) $(\forall a \in \mathbb{R}_+) \sqrt{a^3} = a\sqrt{a}$
 c) $(\forall a \in \mathbb{R}) \sqrt[3]{a^4} = a\sqrt[3]{a}$
 d) $(\forall a \in \mathbb{R}_+)(\forall m \in \mathbb{N})(\forall n \in \mathbb{N}^* \setminus \{1\}) a^{\frac{m}{n}} = \sqrt[n]{a^m}$
 e) $(\forall a \in \mathbb{R}_+) \sqrt{\sqrt{a}} = \sqrt[4]{a}$
 f) $(\forall a \in \mathbb{R}^*)(\forall n \in \mathbb{Z}) \sqrt{a^{2n}} = |a|^n$

Exercice 2

- (1) $\sqrt{36^{\frac{1}{3}}} \cdot \sqrt[3]{24} \cdot \sqrt[4]{16^{-3}} = \left((2^2 \cdot 3^2)^{\frac{1}{3}} \right)^{\frac{1}{2}} \cdot (2^3 \cdot 3)^{\frac{1}{3}} \cdot (2^{-12})^{\frac{1}{4}}$
 $= 2^{\frac{1}{3}} \cdot 3^{\frac{1}{3}} \cdot 2 \cdot 3^{\frac{1}{3}} \cdot 2^{-3}$
 $= 2^{-\frac{5}{3}} \cdot 3^{\frac{2}{3}} = \frac{\sqrt[3]{9}}{2\sqrt[3]{4}} \left(= \frac{\sqrt[3]{9\sqrt{2}}}{4} = \frac{\sqrt[3]{18}}{4} \right)$
- (2) $\frac{\sqrt{a\sqrt[3]{4a}}}{2^{-\frac{1}{2}} \cdot \sqrt[6]{32a}} = \frac{\left(a(2^2 a)^{\frac{1}{3}} \right)^{\frac{1}{2}}}{2^{-\frac{1}{2}} \cdot (2^5 a)^{\frac{1}{6}}} = \frac{a^{\frac{1}{2}} \cdot 2^{\frac{1}{3}} \cdot a^{\frac{1}{6}}}{2^{-\frac{1}{2}} \cdot 2^{\frac{5}{6}} a^{\frac{1}{6}}} = a^{\frac{1}{2}} = \sqrt{a}$
- (3) $\left(2a^{-1} \cdot \sqrt[3]{ab^4} \right)^{\frac{1}{4}} \sqrt[4]{2^{-1}a^2b^{-10}} = 2^{\frac{1}{4}} a^{-\frac{1}{4}} (ab^4)^{\frac{1}{12}} (2^{-1}a^2b^{-10})^{\frac{1}{4}}$
 $= 2^{\frac{1}{4}} a^{-\frac{1}{4}} a^{\frac{1}{12}} b^{\frac{1}{3}} 2^{-\frac{1}{4}} a^{\frac{1}{2}} b^{-\frac{5}{2}}$
 $= a^{\frac{1}{3}} b^{-\frac{13}{6}}$
 $= \frac{\sqrt[3]{a}}{b^2\sqrt[6]{b}}$
- (4) $\frac{\sqrt[4]{25 \cdot a^2 \cdot \sqrt[3]{b^6}}}{10\sqrt{a^{\frac{1}{3}}b^2}} = \frac{5^{\frac{1}{2}} a^{\frac{1}{2}} b^{\frac{1}{2}}}{2 \cdot 5 \cdot a^{\frac{1}{6}} b} = \frac{a^{\frac{1}{3}}}{2 \cdot 5^{\frac{1}{2}} \cdot b^{\frac{1}{2}}} = \frac{\sqrt[3]{a}}{2\sqrt{5b}}$

Exercice 3

$$(1) \quad u^{12} - 25u^4 = 150 - 6u^8$$

$$\Leftrightarrow u^4(u^8 - 25) = 6(25 - u^8)$$

$$\Leftrightarrow u^4(u^8 - 25) - 6(25 - u^8) = 0$$

$$\Leftrightarrow u^4(u^8 - 25) + 6(u^8 - 25) = 0$$

$$\Leftrightarrow (u^8 - 25)(u^4 + 6) = 0$$

$$\Leftrightarrow u^8 = 25 \text{ ou } \underbrace{u^4 = -6}_{\text{impossible}}$$

$$\Leftrightarrow u = \sqrt[8]{25} \text{ ou } u = -\sqrt[8]{25}$$

$$\Leftrightarrow u = \sqrt[4]{5} \text{ ou } u = -\sqrt[4]{5}$$

$$S = \{\pm\sqrt[4]{5}\}$$

$$\begin{aligned} (2) \quad \frac{1}{\sqrt{6}} - \frac{\sqrt{2} + 1}{\sqrt{3} - 1} - 1 &= \frac{\sqrt{6}}{6} - \frac{\sqrt{2} + 1}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} - 1 \\ &= \frac{\sqrt{6}}{6} - \frac{\sqrt{6} + \sqrt{2} + \sqrt{3} + 1}{3 - 1} - 1 \\ &= \frac{\sqrt{6}}{6} - \frac{3\sqrt{6} + 3\sqrt{2} + 3\sqrt{3} + 3}{6} - \frac{6}{6} \\ &= \frac{-2\sqrt{6} - 3\sqrt{2} - 3\sqrt{3} - 9}{6} \\ &= -\frac{\sqrt{6}}{3} - \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} - \frac{3}{2} \end{aligned}$$