

Question 1

$$\begin{aligned}(1) \quad & (3a - 2b + c - 7d)(3a + 2b + c + 7d) \\ &= [(3a + c) - (2b + 7d)][(3a + c) + (2b + 7d)] \\ &= (3a + c)^2 - (2b + 7d)^2 \\ &= 9a^2 + 6ac + c^2 - (4b^2 + 28bd + 49d^2) \\ &= 9a^2 + 6ac + c^2 - 4b^2 - 28bd - 49d^2\end{aligned}$$

$$\begin{aligned}(2) \quad & (-ax - b)^2 \cdot 2 \cdot (b - ax)^2 \\ &= 2 \cdot (ax + b)^2 (b - ax)^2 \\ &= 2 \cdot [(b + ax)(b - ax)]^2 \\ &= 2 \cdot (b^2 - a^2x^2)^2 \\ &= 2 \cdot (b^4 - 2a^2b^2x^2 + a^4x^4) \\ &= 2b^4 - 4a^2b^2x^2 + 2a^4x^4\end{aligned}$$

$$\begin{aligned}(3) \quad & (-3a^2 - y^5)^3 \\ &= -(3a^2 + y^5)^3 \\ &= -\left[(3a^2)^3 + 3 \cdot (3a^2)^2 y^5 + 3 \cdot 3a^2 (y^5)^2 + (y^5)^3\right] \\ &= -(27a^6 + 27a^4y^5 + 9a^2y^{10} + y^{15}) \\ &= -27a^6 - 27a^4y^5 - 9a^2y^{10} - y^{15}\end{aligned}$$

Question 2

$$\begin{aligned}(1) \quad & a^6 - b^6 \\ &= (a^3 - b^3)(a^3 + b^3) \\ &= (a - b)(a^2 + ab + b^2)(a + b)(a^2 - ab + b^2)\end{aligned}$$

$$\begin{aligned}(2) \quad & x^3 - 8y^3 - x^2z + 4y^2z \\ &= (x^3 - 8y^3) - (x^2z - 4y^2z) \\ &= (x - 2y)(x^2 + 2xy + 4y^2) - z(x - 2y)(x + 2y) \\ &= (x - 2y)[(x^2 + 2xy + 4y^2) - z(x + 2y)] \\ &= (x - 2y)(x^2 + 2xy + 4y^2 - xz - 2yz)\end{aligned}$$

$$\begin{aligned}
(3) \quad & (a - 5b)^2 (x + y)^3 + (5b - a)^3 (3x + 3y)^2 \\
& = (a - 5b)^2 (x + y)^3 - 9(a - 5b)^3 (x + y)^2 \\
& = (a - 5b)^2 (x + y)^2 [(x + y) - 9(a - 5b)] \\
& = (a - 5b)^2 (x + y)^2 (x + y - 9a + 45b) \\
(4) \quad & (3a + b)(x - y)^2 + (-12a - 4b)(2x + 3y)^2 \\
& = (3a + b)(x - y)^2 - 4(3a + b)(2x + 3y)^2 \\
& = (3a + b)[(x - y)^2 - 4(2x + 3y)^2] \\
& = (3a + b)[(x - y) - 2(2x + 3y)][(x - y) + 2(2x + 3y)] \\
& = (3a + b)(x - y - 4x - 6y)(x - y + 4x + 6y) \\
& = (3a + b)(-3x - 7y)(5x + 5y) \\
& = -5(3a + b)(3x + 7y)(x + y) \\
(5) \quad & 4ab + 81 - a^2 - 4b^2 \\
& = 81 - (a^2 - 4ab + 4b^2) \\
& = 9^2 - (a - 2b)^2 \\
& = (9 - a + 2b)(9 + a - 2b) \\
(6) \quad & x^2 + 2x + 1 + (x + 1)^3 \\
& = (x + 1)^2 + (x + 1)^3 \\
& = (x + 1)^2 (1 + x + 1) \\
& = (x + 1)^2 (x + 2)
\end{aligned}$$

Bonus

$$\begin{aligned}
& \frac{3x}{5} - \left\{ 2 - \left[x - \left(1 - \frac{x}{2} \right) \right] \right\} = 0 \\
\Leftrightarrow & \frac{3x}{5} - \left[2 - \left(x - 1 + \frac{x}{2} \right) \right] = 0 \\
\Leftrightarrow & \frac{3x}{5} - 2 + \left(x - 1 + \frac{x}{2} \right) = 0 \\
\Leftrightarrow & \frac{3x}{5} - 2 + x - 1 + \frac{x}{2} = 0 \\
\Leftrightarrow & \frac{21x}{10} - 3 = 0 \\
\Leftrightarrow & \frac{21x}{10} = 3 \cdot \frac{10}{21} \\
\Leftrightarrow & x = \frac{10}{7} \\
S = & \left\{ \frac{10}{7} \right\}
\end{aligned}$$