

Question 1

$$(1) \quad x^2 = 5 \Leftrightarrow x = \sqrt{5} \text{ ou } x = -\sqrt{5} \quad S = \{\pm\sqrt{5}\}$$

$$(2) \quad 9x^2 = 4 \Leftrightarrow x^2 = \frac{4}{9} \Leftrightarrow x = \frac{2}{3} \text{ ou } x = -\frac{2}{3} \quad S = \left\{\pm\frac{2}{3}\right\}$$

$$(3) \quad x^2 + 7 = 0 \quad \text{impossible} \quad S = \emptyset$$

$$(4) \quad x^4 = 81 \Leftrightarrow x^2 = 9 \text{ ou } \underbrace{x^2 = -9}_{\text{impossible}} \Leftrightarrow x = \pm 3 \quad S = \{\pm 3\}$$

$$(5) \quad x^2 - 5x - 6 = 0$$

$$\Leftrightarrow x^2 + x - 6x - 6 = 0$$

$$\Leftrightarrow x(x+1) - 6(x+1) = 0 \quad S = \{-1, 6\}$$

$$\Leftrightarrow (x+1)(x-6) = 0$$

$$\Leftrightarrow x = -1 \text{ ou } x = 6$$

$$(6) \quad (2x+3)^2 - 25(x-1)^2 = 0$$

$$\Leftrightarrow [(2x+3) - 5(x-1)][(2x+3) + 5(x-1)] = 0$$

$$\Leftrightarrow (2x+3-5x+5)(2x+3+5x-1) = 0$$

$$\Leftrightarrow (-3x+8)(7x-2) = 0 \quad S = \left\{\frac{8}{3}, -\frac{2}{7}\right\}$$

$$\Leftrightarrow x = \frac{8}{3} \text{ ou } x = \frac{2}{7}$$

$$(7) \quad (x+3)(x^2-4)(-5x+12) = 0$$

$$\Leftrightarrow x = -3 \text{ ou } x^2 = 4 \text{ ou } -5x = -12$$

$$\Leftrightarrow x = -3 \text{ ou } x = \pm 2 \text{ ou } x = \frac{12}{5} \quad S = \left\{-3, \pm 2, \frac{12}{5}\right\}$$

$$(8) \quad (x-2)(x^2-3x) = -(2-x)(x-4)$$

$$\Leftrightarrow (x-2)(x^2-3x) + (2-x)(x-4) = 0$$

$$\Leftrightarrow (x-2)(x^2-3x) - (x-2)(x-4) = 0$$

$$\Leftrightarrow (x-2)(x^2-3x-x+4) = 0$$

$$\Leftrightarrow (x-2)(x^2-4x+4) = 0 \quad S = \{2\}$$

$$\Leftrightarrow (x-2)(x-2)^2 = 0$$

$$\Leftrightarrow (x-2)^3 = 0 \Leftrightarrow x-2 = 0$$

$$\Leftrightarrow x = 2$$

Question 2

$$(1) \quad \frac{1}{2x} + \frac{3}{x-2} = \frac{x-1}{x^2-2x}$$

$$\Leftrightarrow \frac{1}{2x} + \frac{3}{x-2} = \frac{x-1}{x(x-2)}$$

C.E. : $x \neq 0$ et $x \neq 2$

$$\Leftrightarrow \frac{x-2}{2x(x-2)} + \frac{6x}{2x(x-2)} = \frac{2x-2}{2x(x-2)} \quad / \cdot 2x(x-2)$$

$$\Leftrightarrow x-2+6x=2x-2$$

$$\Leftrightarrow 5x=0$$

$$\Leftrightarrow x=0 \text{ à exclure !}$$

$$S = \emptyset$$

$$(2) \quad \frac{x-3}{x^2-1} - \frac{x}{1-x} = 2$$

$$\Leftrightarrow \frac{x-3}{(x-1)(x+1)} + \frac{x}{x-1} = \frac{2(x^2-1)}{(x-1)(x+1)}$$

C.E. : $x \neq 1$ et $x \neq -1$

$$\Leftrightarrow \frac{x-3}{(x-1)(x+1)} + \frac{x(x+1)}{(x-1)(x+1)} = \frac{2(x^2-1)}{(x-1)(x+1)}$$

$$\Leftrightarrow x-3+x^2+x=2x^2-2$$

$$\Leftrightarrow x^2+2x-3-2x^2+2=0$$

$$\Leftrightarrow -x^2+2x-1=0 \quad / \cdot (-1)$$

$$\Leftrightarrow x^2-2x+1=0$$

$$\Leftrightarrow (x-1)^2=0$$

$$\Leftrightarrow x=1 \text{ à exclure !}$$

$$S = \emptyset$$

$$(3) \quad \frac{1-3x}{x^2+x} = \frac{x-3}{x+1}$$

$$\Leftrightarrow \frac{1-3x}{x(x+1)} = \frac{x-3}{x+1}$$

C.E. : $x \neq 0$ et $x \neq -1$

$$\Leftrightarrow \frac{1-3x}{x(x+1)} = \frac{x^2-3x}{x(x+1)}$$

$$\Leftrightarrow 1-3x=x^2-3x$$

$$\Leftrightarrow x^2=1$$

$$\Leftrightarrow x=-1 \text{ (à exclure) ou } x=1$$

$$S = \{1\}$$

Question 3

(1) Comme $\frac{1}{A} = \frac{1}{\frac{1}{x}} = x$, on a :

$$\begin{aligned} & \frac{1}{A} + \frac{1}{B} + \frac{1}{C} \\ &= x + \frac{1}{x+1} + \frac{1}{x^2} \\ &= \frac{x^3(x+1)}{x^2(x+1)} + \frac{x^2}{x^2(x+1)} + \frac{x+1}{x^2(x+1)} \\ &= \frac{x^4 + x^3 + x^2 + x + 1}{x^2(x+1)} \end{aligned}$$

(2) Comme $\frac{A}{B} = \frac{\frac{1}{x}}{x+1} = \frac{1}{x} \cdot \frac{1}{x+1} = \frac{1}{x(x+1)}$, on a :

$$\begin{aligned} & \frac{\frac{A}{B}}{C} \\ &= \frac{\frac{1}{x(x+1)}}{x^2} \\ &= \frac{1}{x(x+1)} \cdot \frac{1}{x^2} \\ &= \frac{1}{x^3(x+1)} \end{aligned}$$

G. Lorang