

Durée : 60'

Calculatrice non autorisée

Question 1

9 (=3+3+3) points

Montrez comment on peut faire mentalement (donc de la manière la plus simple possible) les calculs suivants :

$$A = 998^2 ; \quad B = 1997 \cdot 2003 ; \quad C = 4,58^2 + 2 \cdot 4,58 \cdot 0,42 + 0,42^2$$

$$\begin{aligned} A &= (1000 - 2)^2 \\ &= 1'000'000 - 2 \cdot 1000 \cdot 2 + 2^2 \\ &= 1'000'000 - 4'000 + 4 \\ &= 996'004 \\ B &= 1997 \cdot 2003 = (2000 - 3) \cdot (2000 + 3) \\ &= 4'000'000 - 9 \\ &= 3'999'991 \\ C &= 4,58^2 + 2 \cdot 4,58 \cdot 0,42 + 0,42^2 \\ &= (4,58 + 0,42)^2 \\ &= 5^2 = 25 \end{aligned}$$

Question 2

6 (=3+3) points

Calculer et simplifier $\frac{\frac{A}{B}}{C}$ et $\frac{B}{\frac{C}{A}}$ sachant que $A = x + 2$, $B = \frac{1}{x - 2}$ et $C = x^2 - 4$.

$$\frac{\frac{A}{B}}{C} = \frac{\frac{x+2}{\frac{1}{x-2}}}{x^2-4} = \frac{\frac{x+2}{1} \cdot \frac{x-2}{1}}{x^2-4} = \frac{(x+2)(x-2)}{x^2-4} = 1$$

$$\frac{\frac{B}{C}}{\frac{C}{A}} = \frac{\frac{1}{x-2}}{\frac{x^2-4}{x+2}} = \frac{1}{x-2} \cdot \frac{x+2}{x^2-4} = \frac{1}{x-2} \cdot \frac{x+2}{(x-2)(x+2)} = \frac{1}{(x-2)^2}$$

Question 3

32 (=7+9+8+8) points

Calculer et simplifier autant que possible.

$$(1) \quad A = \frac{2x^2 + 2x}{10x^2 - 15x} - \frac{2x - 3}{9 - 12x + 4x^2}$$

$$\begin{aligned}
 A &= \frac{\cancel{2x}(x+1)}{5x(2x-3)} - \frac{2x-3}{(3-2x)^2} \\
 &= \frac{2(x+1)}{5(2x-3)} - \frac{\cancel{2x-3}}{(2x-3)^2} \\
 &= \frac{2(x+1)}{5(2x-3)} - \frac{5}{5(2x-3)} \\
 &= \frac{2x+2-5}{5(2x-3)} \\
 &= \frac{\cancel{2x-3}}{5(\cancel{2x-3})} = \frac{1}{5}
 \end{aligned}$$

$$(2) \quad B = \frac{x+4}{2x^3 - 4x^2} - \frac{x-4}{2x^3 + 4x^2} + \frac{x+4}{4-x^2}$$

$$\begin{aligned}
 B &= \frac{x+4}{2x^2(x-2)} - \frac{x-4}{2x^2(x+2)} + \frac{x+4}{(2-x)(x+2)} \\
 &= \frac{(x+4)(x+2)}{2x^2(x-2)(x+2)} - \frac{(x-4)(x-2)}{2x^2(x+2)(x-2)} - \frac{(x+4) \cdot 2x^2}{2x^2(x-2)(x+2)} \\
 &= \frac{x^2+4x+2x+8 - (x^2-2x-4x+8) - 2x^3-8x^2}{2x^2(x+2)(x-2)} \\
 &= \frac{\cancel{x^2}+6x+8 - \cancel{x^2}+6x-8 - 2x^3-8x^2}{2x^2(x+2)(x-2)} \\
 &= \frac{-2x^3-8x^2+12x}{2x^2(x+2)(x-2)} = \frac{\cancel{-2x}(x^2+4x-6)}{2x^2(x+2)(x-2)} \\
 &= -\frac{x^2+4x-6}{x(x+2)(x-2)}
 \end{aligned}$$

$$(3) \quad C = \frac{54 - 6x^2}{10x - 35x^2} : \frac{3x^2 - 18x + 27}{49x^2 - 28x + 4}$$

$$\begin{aligned} C &= \frac{6(9-x^2)}{5x \cdot (2-7x)} : \frac{3(x^2-6x+9)}{(7x-2)^2} \\ &= \frac{\overset{2}{\cancel{6}} \cdot (3-x)(3+x)}{5x \cdot (2-7x)} \cdot \frac{(7x-2)^2}{\underset{1}{\cancel{3}} \cdot (x-3)^2} \\ &= \frac{2 \cdot \cancel{(3-x)} \cdot (3+x)}{5x \cdot \cancel{(2-7x)}} \cdot \frac{(2-7x)^2}{(3-x)^2} \\ &= \frac{2 \cdot (3+x)(2-7x)}{5x(3-x)} \end{aligned}$$

$$(4) \quad D = \frac{\frac{6b-3a}{ab^2}}{\frac{1}{b} - \frac{4}{a} + \frac{4b}{a^2}}$$

$$\begin{aligned} D &= \frac{\frac{3(2b-a)}{ab^2}}{\frac{a^2}{a^2b} - \frac{4ab}{a^2b} + \frac{4b^2}{a^2b}} \\ &= \frac{\frac{3(2b-a)}{ab^2}}{\frac{a^2 - 4ab + 4b^2}{a^2b}} \\ &= \frac{3 \cdot \overset{-1}{\cancel{(2b-a)}} \cdot \overset{-1}{\cancel{a^2b}}}{\cancel{ab^2} \cdot (a-2b)^2} = \frac{-3a}{b(a-2b)} \end{aligned}$$

Question 4

13 (=5+8) points

(1) Calculer et simplifier $E = \frac{\frac{1}{x} - x}{\frac{(x+1)^2}{x^2}}$.

$$E = \frac{\frac{1-x^2}{x}}{\frac{(x+1)^2}{x^2}} = \frac{\frac{(1-x)(1+x)}{x}}{\frac{(x+1)^2}{x^2}}$$

$$= \frac{\cancel{(1-x)}\cancel{(1+x)}}{\cancel{x}} \cdot \frac{x^2}{(x+1)^2} = \frac{x(1-x)}{x+1}$$

(2) Calculer ensuite E a) si $x = 3^{-2}$ et b) si $x = -\frac{3}{5}$.

a) si $x = 3^{-2}$ alors

$$E = \frac{3^{-2} \cdot (1 - 3^{-2})}{3^{-2} + 1} = \frac{\frac{1}{9} \cdot (1 - \frac{1}{9})}{\frac{1}{9} + 1}$$

$$= \frac{\frac{1}{9} \cdot \frac{8}{9}}{\frac{10}{9}} = \frac{\frac{8}{81} \cdot \frac{9}{10}}{\frac{10}{9}} = \frac{4}{45}$$

b) si $x = -\frac{3}{5}$ alors $E = \frac{-\frac{3}{5} \cdot (1 + \frac{3}{5})}{-\frac{3}{5} + 1}$

$$= \frac{-\frac{3}{5} \cdot \frac{8}{5}}{\frac{2}{5}} = -\frac{\frac{24}{25} \cdot \frac{5}{2}}{\frac{2}{5}} = -\frac{12}{5}$$

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