1) $z_0 = a \in \mathbb{R}$

$a^4 + (-6+4i)a^3 + (-2-5i)a^2 + (7-35i)a - 42 - 18i = 0$

$\iff (a^4 - 6a^3 - 2a^2 + 7a - 42) + (4a^3 - 5a^2 - 35a - 18) = 0$

$\iff \begin{cases} a^4 - 6a^3 - 2a^2 + 7a - 42 = 0 & (1) \\ \text{et} \\ 4a^3 - 5a^2 - 35a - 18 = 0 & (2) \end{cases}$

Résoudre (2) $\quad P(1) \neq 0$; $P(-1) \neq 0$; $P(2) = 32 - 20 - 70 - 18 \neq 0$

$P(-2) = -32 - 20 + 70 - 18 = 0$

Dans (1) $\quad a = -2 \to 16 + 48 - 8 - 14 - 42 = 0$

Donc $z = -2$ est une solution

	1	$-6+4i$	$-2-5i$	$7-35i$	$-42-18i$
-2		-2	$16-8i$	$-28+26i$	$42+18i$
	1	$-8+4i$	$14-13i$	$-21-9i$	0

$(z+2)\left[z^3 + (-8+4i)z^2 + (14-13i)z + (-21-9i)\right] = 0$

$z = bi$

$\to -b^3 i - b^2(-8+4i) + bi(14-13i) + (-21-9i) = 0$

$\iff -b^3 i + 8b^2 - 4b^2 i + 14bi + 13b\cancel{i} - 21 - 9i = 0$

$\iff (8b^2 + 13b - 21) + (-b^3 - 4b^2 + 14b - 9)i = 0$

$\iff \begin{cases} 8b^2 + 13b - 21 = 0 \\ -b^3 - 4b^2 + 14b - 9 = 0 \end{cases} \qquad \Delta = 29^2 \quad b_{1,2} = \dfrac{-13 \pm 29}{16} \to -\dfrac{42}{16}$; $\to 1 \to$ dans (2) $= 0$

$z = i$ est une racine

	1	$-8+4i$	$14-13i$	$-21-9i$
i		i	$-8i-5$	$9i+21$
	1	$-8+5i$	$9-21i$	0

$(z+2)(z-i)\left[z^2 + (-8+5i)z + (9-21i)\right] = 0$

$\Delta = (-8+5i)^2 - 4(9-21i) = 64 - 80i - 25 - 36 + 84i = 3 + 4i$

$\left.\begin{array}{l} a^2 - b^2 = 3 \\ 2ab = 4 \\ a^2 + b^2 = 5 \end{array}\right\} \Rightarrow \begin{array}{l} 2a^2 = 8 \\ a^2 = 4 \end{array}$

$a = 2 \Rightarrow b = 1$

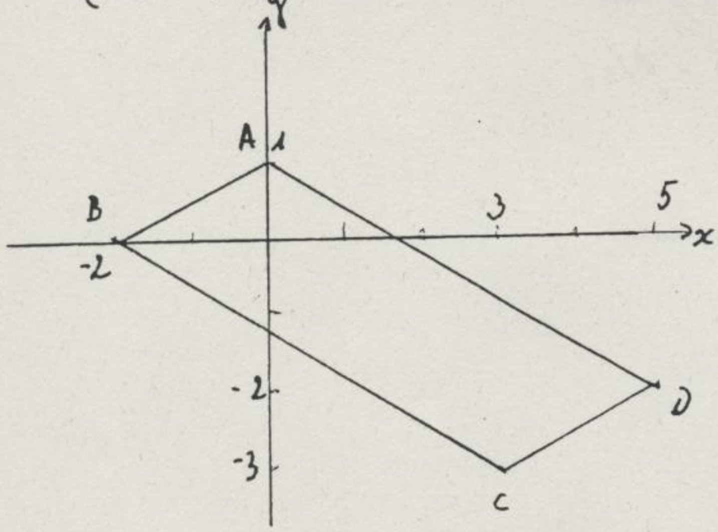
$a = -2 \Rightarrow b = -1$

$\Rightarrow \Delta = (2+i)^2$

$$z_{1,2} = \frac{8-5i \pm (2+i)}{2} \nearrow \frac{8-5i+2+i}{2} = 5-2i$$

$$\searrow \frac{8-5i-2-i}{2} = 3-3i$$

$S = \{2 ; i ; 5-2i ; 3-3i\}$



$A(i)$
$B(-2)$
$C(3-3i)$
$D(5-2i)$

$\vec{AD}(z_D - z_A) \rightarrow \vec{AD}(5-2i-i)$

$\vec{AD}(5-3i)$

$\vec{BC}(z_C - z_B) \rightarrow \vec{BC}(3-3i+2)$

$\vec{BC}(5-3i)$

Donc (A,B,C,D) est un parallélogramme

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1) a) $\dfrac{4 \cdot 31!}{32!} = \dfrac{4}{32} = \dfrac{1}{8} = 0,125$

$$\frac{A_{28}^{9} \cdot 4 \cdot 22!}{32!} = \frac{\frac{28!}{19!} 4 \cdot 22!}{32!} = \frac{28!\, 4 \cdot 22!}{19!\, 32!} \simeq 0,043$$

b) $X : 0 ; 1 ; 2 ; 3 ;$

$$P(X=0) = \frac{C_{16}^{3}}{C_{20}^{3}} = \frac{560}{1140} \simeq 0,4912$$

$$P(X=1) = \frac{C_{4}^{1} C_{16}^{2}}{C_{20}^{3}} = \frac{480}{1140} \simeq 0,4211$$

$$P(X=2) = \frac{C_{4}^{2} C_{16}^{1}}{C_{20}^{3}} = \frac{96}{1140} \simeq 0,0842$$

$$P(X=3) = \frac{C_{4}^{3}}{C_{20}^{3}} = \frac{4}{1140} \simeq 0,0035$$

c) 3 ou 6 $\longrightarrow$ +10 € $\quad p = 1/3$

1,2,4,5 $\longrightarrow$ -6 € $\quad p = 2/3$

$P(X = +30) = \left(\frac{1}{3}\right)^3 = 1/27$

$P(X = 20-6 = 14) = \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right) \cdot 3 = 6/27$

$P(X = 10-12 = -2) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 \cdot 3 = 12/27$

$P(X = -18) = \left(\frac{2}{3}\right)^3 = 8/27$

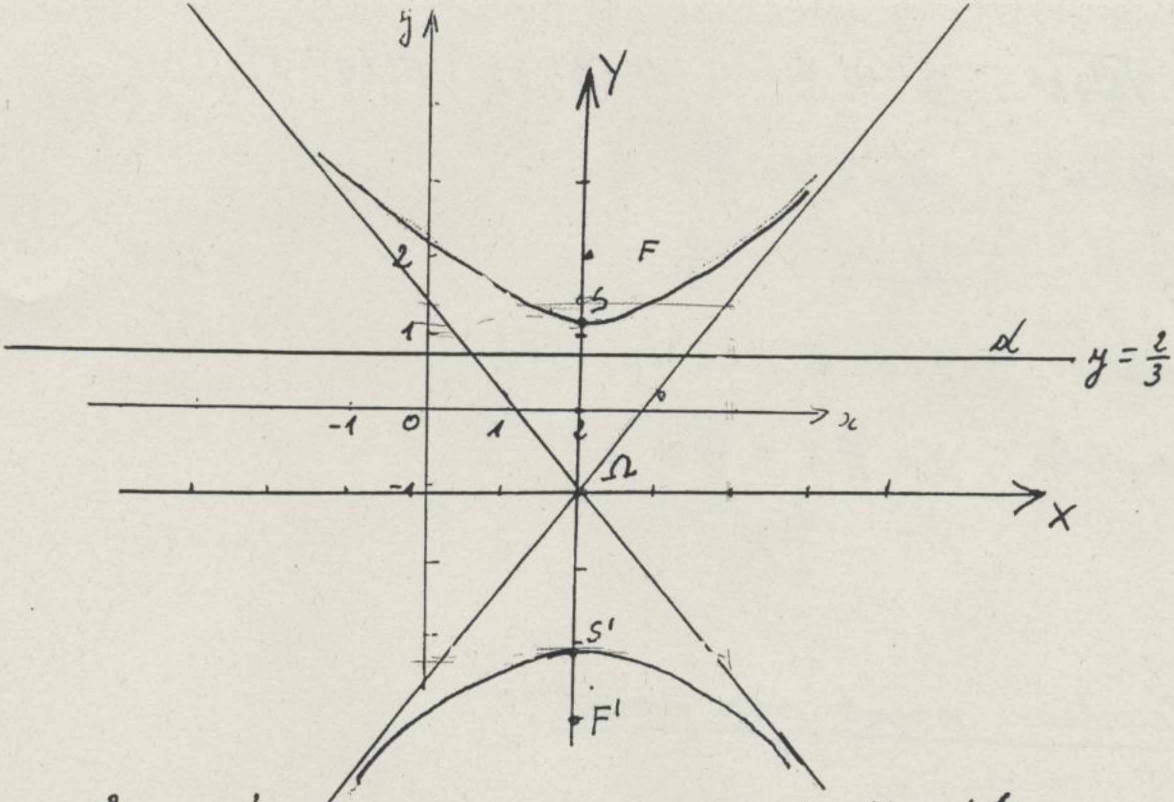
x_i	p_i	$x_i p_i$	$p_i (x_i - E)^2$
30	1/27	30/27	$\frac{1024}{27}$
14	4/27	84/27	$\frac{1536}{27}$
-2	12/27	-24/27	$\frac{0}{27}$
-18	8/27	-144/27	$\frac{2048}{27}$
	$\Sigma = 1$	$E = -2$	$\frac{4608}{27}$

$E(X) = -2$

$V(X) = \frac{4608}{27} \simeq 171$

$\sigma(X) \simeq 13$

3)



$e > 1 \longrightarrow$ Hyperbole

$P(x,y) \in \Gamma$

$d^2(P,F) = (x-2)^2 + (y-2)^2$

$d^2(P,d) = \left(y - \frac{2}{3}\right)^2$

$P \in \Gamma \Leftrightarrow d^2(P,F) = \frac{9}{5} d^2(P,d)$

$\Leftrightarrow x^2 - 4x + 4 + y^2 - 4y + 4 = \frac{9}{5}\left(y^2 - \frac{4}{3}y + \frac{4}{9}\right)$

$\Leftrightarrow x^2 - 4x + y^2 - 4y + 8 = \frac{9}{5}y^2 - \frac{12}{5}y + \frac{4}{5}$

$\Leftrightarrow x^2 - 4x \cancel{+} - \frac{4}{5}y^2 - \frac{8}{5}y + 8 - \frac{4}{5} = 0$

$\Leftrightarrow x^2 - 4x - \frac{4}{5}(y^2 + 2y) + 8 - \frac{4}{5} = 0$

$\Leftrightarrow (x^2 - 4x + 4) - \frac{4}{5}(y^2 + 2y + 1) + 8 - \frac{4}{5} + \frac{4}{5} - 4 = 0$

$\Leftrightarrow (x-2)^2 - \frac{4}{5}(y+1)^2 + 4 = 0$

$\Leftrightarrow (x-2)^2 - \frac{4}{5}(y+1)^2 = -4 \quad |:(-4)$

$\Leftrightarrow -\frac{(x-2)^2}{4} + \frac{(y+1)^2}{5} = 1$ H de centre $\Omega(2,-1)$ d'axe $\parallel Oy$

Dans $(\Omega, \vec{i}, \vec{j})$

$x - 2 = X$

$y + 1 = Y$

$-\frac{X^2}{4} + \frac{Y^2}{5} = 1$

$c = \sqrt{a^2+b^2} = 3$ $\qquad F(0\,;3) \quad F'(0\,;-3)$

Directrices : $\quad d_F \equiv y = \frac{b^2}{c} = \frac{5}{3}$

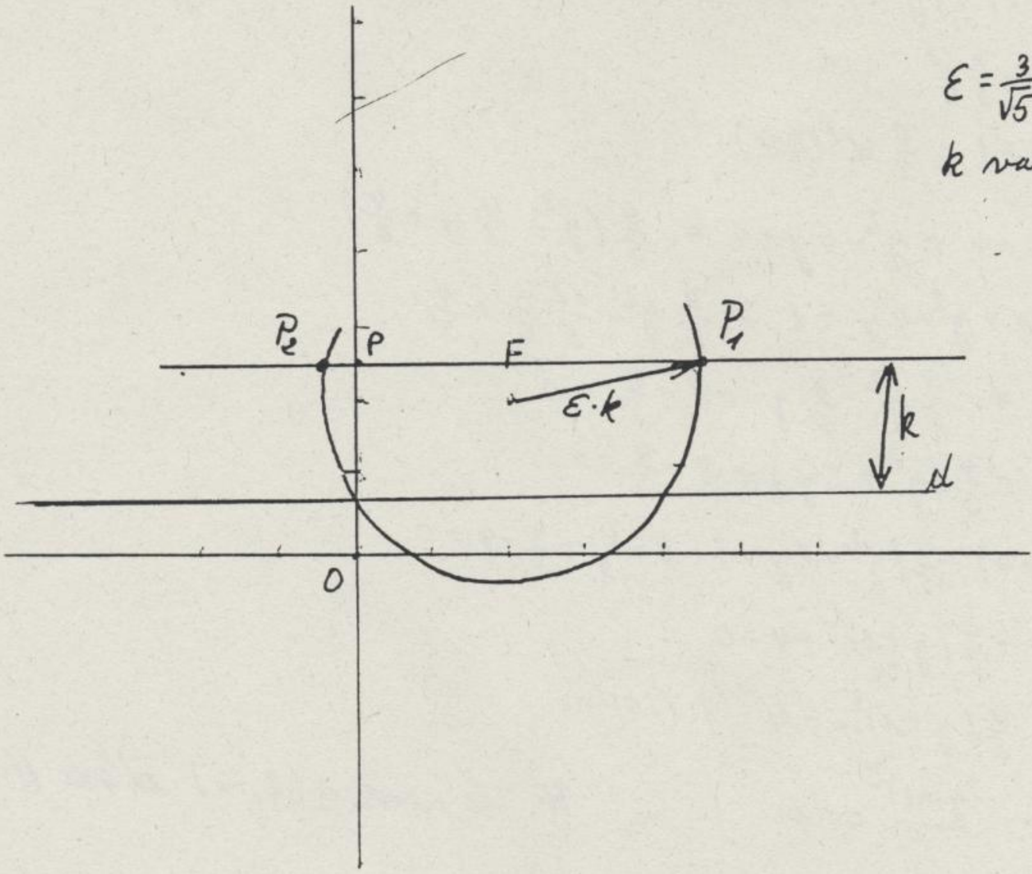
$\qquad d_{F'} \equiv y = -\frac{5}{3}$

Sommets $\quad S(0\,;\sqrt{5}) \quad S'(0\,;-\sqrt{5})$

Asymptotes : $\quad y = \frac{b}{a}x = \frac{\sqrt{5}}{2}x$

$\qquad y = -\frac{\sqrt{5}}{2}x$

Construction point par point :



$\varepsilon = \frac{3}{\sqrt{5}}$

k varie

4)

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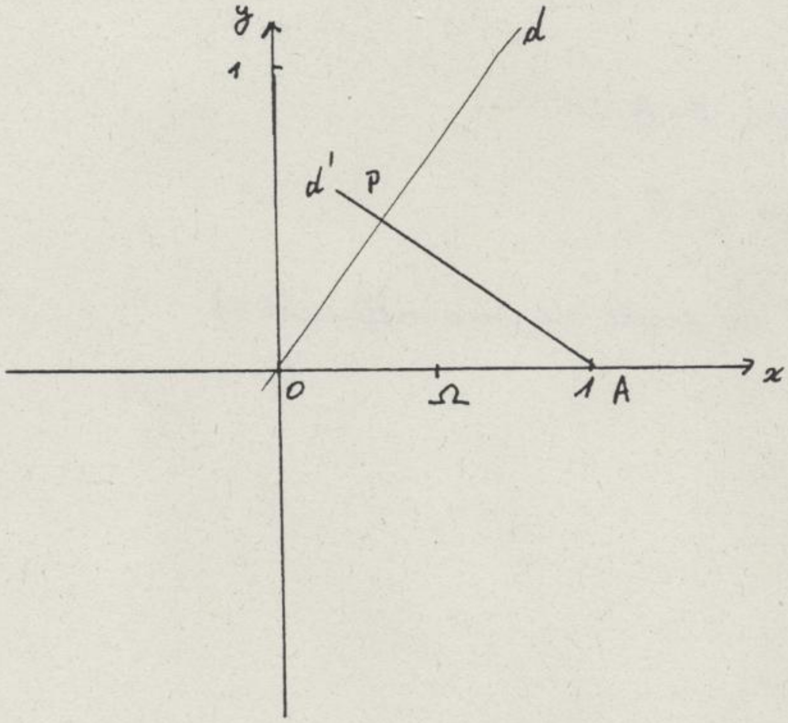

$d \equiv y = mx \qquad (d \neq Oy)$

$d' \equiv y = -\frac{1}{m}x + b \qquad$ passe par A : $0 = -\frac{1}{m} + b \Rightarrow b = \frac{1}{m}$

$y = -\frac{1}{m}x + \frac{1}{m} \qquad m \neq 0$

$d \cap d'$: $mx = -\frac{1}{m}x + \frac{1}{m}$

$\Leftrightarrow (m + \frac{1}{m})x = \frac{1}{m}$

$\Leftrightarrow \frac{m^2+1}{m}x = \frac{1}{m}$

$\Leftrightarrow x = \frac{1}{m^2+1} \Rightarrow y = mx = \frac{m}{m^2+1} \qquad P\left(\frac{1}{m^2+1} ; \frac{m}{m^2+1}\right)$

$x \neq 0$

$y = \frac{m}{m^2+1} = m \cdot \frac{1}{m^2+1} = m \cdot x \Rightarrow m = \frac{y}{x}$

$x = \frac{1}{m^2+1} = \frac{1}{\frac{y^2}{x^2}+1} = \frac{1}{\frac{y^2+x^2}{x^2}} = \frac{x^2}{y^2+x^2}$

$\Rightarrow x(y^2+x^2) = x^2 \Rightarrow x(y^2+x^2-x) = 0 \Rightarrow x\left(x^2 - x + \frac{1}{4} + y^2 - \frac{1}{4}\right) = 0$

$\Leftrightarrow x\left[(x-\frac{1}{2})^2 + y^2 - \frac{1}{4}\right] = 0$

$\Leftrightarrow (x - 1/2)^2 + y^2 = \frac{1}{4}$

Cercle de centre $\Omega(1/2, 0)$ et de rayon $1/2$.

= Cercle de diamètre [OA] sauf O ni A

si $m = 0$ alors $P = A$

si $d = Oy$ alors $P = O$

donc le lieu est le cercle de diamètre $[OA]$