(I) 1)- $D_f = [-2;2]$ $\qquad D_{f'} =]-2;2[= D_{f''}$

• $\forall x \in D_f$: $f'(x) = -\frac{f(x)}{\sqrt{4-x^2}} \Rightarrow f \searrow$ sur $[-2;2]$

• $f'_d(\pm 2) = \lim_{x\to 2^\mp} \frac{f(x)-f(\pm2)}{x-(\pm2)} \overset{H}{=} \lim_{x\to2^\mp} \frac{-f(x)}{\sqrt{4-x^2}} = -\infty \Rightarrow$ pl.t // (Oy)

• $\forall x\in D_{f''}$ $\quad f''(x) = -\frac{f'(x)\sqrt{4-x^2} + f(x)\cdot\frac{x}{\sqrt{4-x^2}}}{4-x^2} = f(x)\frac{\sqrt{4-x^2}-x}{(4-x^2)^{3/2}}$

$f''(x)\le 0 \iff \sqrt{4-x^2}\le x \iff x\ge\sqrt2$

x	-2		$\sqrt2$		2
f''		$+$	0	$-$	

p.I. $(\sqrt2; e^{\frac{\pi}{4}}) \simeq 2.2$

2) pt. de tangence $T(x_0,y_0)$

(t): $y = f'(x_0)(x-x_0)+y_0$

$= \frac{-f(x_0)}{\sqrt{4-x_0^2}}(x-x_0)+f(x_0)$

$= -\frac{f(x_0)}{\sqrt{4-x_0^2}}x + f(x_0)\left(\frac{x_0}{\sqrt{4-x_0^2}}+1\right)$

$O\in(t)$: $\frac{x_0}{\sqrt{4-x_0^2}}+1=0 \iff \sqrt{4-x_0^2}=-x_0 \iff x_0=-\sqrt2$

$y_0 = e^{\frac{3\pi}{4}}$

(t): $y = \frac{-e^{3\pi/4}}{\sqrt2}x \simeq -7,46x$

3) $\frac{V}{\pi} = \int_{-2}^{2} e^{2\arccos\frac x2}dx \qquad u = e^{2\arccos\frac x2} \quad v'=1$

$u' = \frac{-2u}{\sqrt{4-x^2}} \quad v=x$

$= x\cdot e^{2\arccos\frac x2}\Big|_{-2}^{2} + 2\int_{-2}^{2}\frac{e^{2\arccos\frac x2}\cdot x}{\sqrt{4-x^2}}dx$

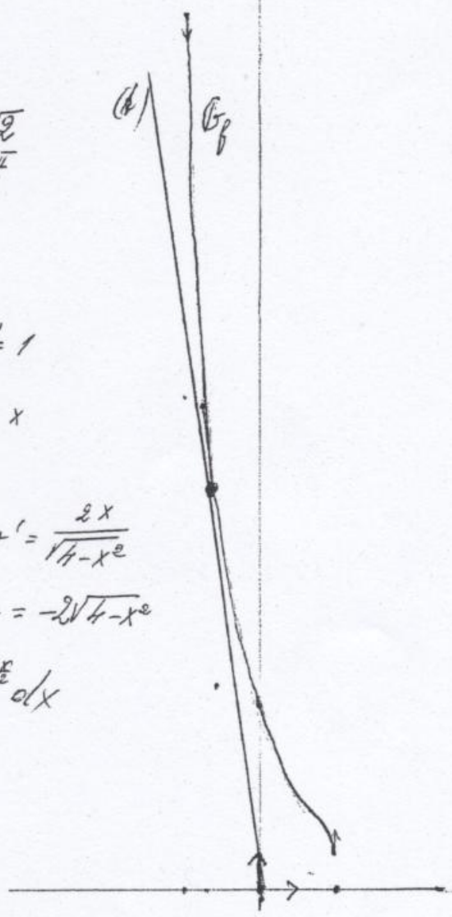
$u = e^{2\arccos\frac x2} \quad v' = \frac{2x}{\sqrt{4-x^2}}$

$u' = \frac{-2u}{\sqrt{4-x^2}} \quad v = -2\sqrt{4-x^2}$

$= 2+2e^{2\pi} \underbrace{-2e^{2\arccos\frac x2}\sqrt{4-x^2}\Big|_{-2}^{2}}_{0} - 4\int_{-2}^2 e^{2\arccos\frac x2}dx$

$\frac{5V}{\pi} = 2(1+e^{2\pi})$

$V = \frac{2\pi}{5}(1+e^{2\pi}) \simeq 674$ u.V.



(II) 1) $D_f =]-1;1[= D_{f'} = D_{f''}$ (cond : $1-x^2>0$)

$\lim\limits_{x\to \pm 1} f(x) = -\infty$ AV: $x = \pm 1$

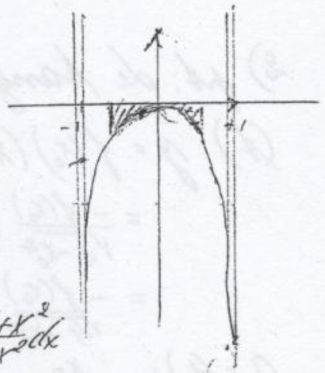
$\forall x \in]-1;1[$ $\quad f(x) = \frac{1}{2}\ln(1-x^2) - \frac{1}{2}\ln(1+x^2)$

$f'(x) = \frac{-x}{1-x^2} - \frac{x}{1+x^2} = \frac{-2x}{1-x^4}$

x	-1		0		1
f'		$+$	0	$-$	
f	$-\infty$	$\nearrow$	0	$\searrow$	$-\infty$

$f''(x) = \frac{(1-x^4)(-2)-(-2x)(-4x^3)}{(1-x^4)^2}$

$= \frac{-2-6x^4}{(1-x^4)^2} = -2\,\frac{1+3x^4}{(1-x^4)^2} < 0$



$\Rightarrow f$ concave

2) f pair C_f sym : $A = 2\int_0^{\frac{1}{2}} -\frac{1}{2}\ln\frac{1-x^2}{1+x^2}\,dx$

$= -\int_0^{1/2} \ln\frac{1-x^2}{1+x^2}\,dx = \int_0^{1/2} \ln\frac{1+x^2}{1-x^2}\,dx$

$u(x) = \ln\frac{1+x^2}{1-x^2}$ $\quad v'(x) = 1$

$u'(x) = \frac{-4x}{x^4-1}$ $\quad v(x) = x$

$= x\ln\frac{1+x^2}{1-x^2}\Big|_0^{\frac{1}{2}} + \int_0^{1/2} \frac{4x^2}{x^4-1}\,dx$

$= \frac{1}{2}\ln\frac{5}{3} + \left(2\,\mathrm{Arctg}\,x + \ln\left|\frac{x-1}{x+1}\right|\right)\Big|_0^{\frac{1}{2}}$

$= \frac{1}{2}\ln\frac{5}{3} + 2\,\mathrm{Arctg}\,\frac{1}{2} + \ln\frac{1}{3}$

$= \frac{1}{2}\ln\frac{5}{27} + 2\,\mathrm{Arctg}\,\frac{1}{2} \simeq 0{,}08$ u. A.

(III) 1) $\int \frac{1-\cos x}{1+\cos x}\,dx$ $\quad \cos x \neq -1 \iff x \neq \pi + k2\pi$

$I =]-\pi;\pi[$

$= \int \frac{2\,tg^2\frac{x}{2}}{2}\,dx$ $\quad \cos x = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}$ (défini sur $]-\pi;\pi[$)

$= \int \left(tg^2\frac{x}{2}+1-1\right)dx$

$= 2\,tg\frac{x}{2} - x + k$

2) $J = \int e^{-2x}\,\operatorname{arctg} e^x\,dx$ $\quad I = \mathbb{R}$

$= -\frac{1}{2}\frac{\operatorname{arctg} e^x}{e^{2x}} + \frac{1}{2}\int \frac{1}{e^x(1+e^{2x})}\,dx$ $\quad u = \operatorname{arctg} e^x \quad v' = e^{-2x}$

$u' = \frac{e^x}{1+e^{2x}} \quad v = -\frac{1}{2}e^{-2x}$

Poser $e^x = u \quad dx = \frac{du}{u}$

$\int \frac{1}{e^x(1+e^{2x})}\,dx = \int \frac{1}{u^2(1+u^2)}\,du = \int\left(\frac{1}{u^2} - \frac{1}{1+u^2}\right)du = -\frac{1}{u} - \operatorname{arctg} u$

$J = -\frac{1}{2}\operatorname{arctg} e^x \cdot e^{-2x} - \frac{1}{2}e^{-x} - \frac{1}{2}\operatorname{arctg} e^x + k$

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1) $\left.\begin{array}{l} x>0 \\ x\neq e\end{array}\right\} \Rightarrow Dom =]0;e[\cup]e;+\infty[$

$1+\frac{1}{2}\ln x = \frac{1}{1-\ln x}$

$\iff (2+\ln x)(1-\ln x) = 2$

$\iff \ln^2 x + \ln x = 0$

$\iff \ln x = 0$ ou $\ln x = -1$

$\iff x = 1$ ou $x = e^{-1}$ $\qquad S = \{1; e^{-1}\}$

2) Poser $e^{3x} = u$: $u^3 - 3u^2 + 4 > 0$

$\iff \underbrace{(u+1)}_{>0}(u^2-4u+4) > 0$

$u \in]-\infty;2[\cup]2;+\infty[$

$x \in]-\infty;\frac{1}{3}\ln 2[\cup]\frac{1}{3}\ln 2;+\infty[$

$S =]-\infty;\frac{1}{3}\ln 2[\cup]\frac{1}{3}\ln 2;+\infty[$

$\lim\limits_{x\to+\infty} \frac{\ln(e^x+1)}{\ln(e^x-1)} = \lim\limits_{x\to+\infty} \frac{x + \overbrace{\ln(1+e^{-x})}^{\to 0}}{x + \underbrace{\ln(1-e^{-x})}_{\to 0}} = 1$

$\lim\limits_{x\to+\infty} \underbrace{\frac{1}{x}}_{\to 0}\,\underbrace{\frac{\ln(e^x+1)}{\ln(e^x-1)}}_{\to 1} = 0$

IV

A) ① $f(x) = ax + b + c.\sin x + d.\cos x$ avec $0 \le x \le \pi$

Conditions : | 1) $f(0) = r$ | 3) $f'(0) = 0$
| 2) $f(\pi) = \frac{r}{3}$ | 4) $f'(\pi) = 0$

Par V200 : $f(x) = \frac{2r}{3} + \frac{r}{3}\cos x = \frac{r}{3}(2 + \cos x)$

② $f''(x) = 0 \iff -\frac{r}{3}\cos x = 0 \iff \cos x = 0 \iff x = \frac{\pi}{2}$ (car $0 \le x \le \pi$)

et $f\left(\frac{\pi}{2}\right) = \frac{2r}{3}$ $I\left(\frac{\pi}{2} ; \frac{2r}{3}\right)$

B) Eq. cart. du quart de cercle : $x^2 + y^2 = r^2$ avec $\begin{cases} -r \le x \le 0 \\ y \ge 0 \end{cases}$

① $V(r) = V_{◢} + V_{▭}$

$$V(r) = \pi \cdot \int_{-r}^{0} (r^2 - x^2)\,dx + \pi \cdot \int_{0}^{\pi} [f(x)]^2\,dx = \frac{2\pi r^3}{3} + \frac{\pi^2 r^2}{2} = \frac{\pi r^2(4r + 3\pi)}{6} \text{ (uv)}$$

② $V(r) = \frac{9\pi(2+\pi)}{8} \overset{V_{200}}{\iff} r = \boxed{\frac{3}{2}} = 1,5$

C) Le raccordement "sans heurts" exige :
①
| 1) $P(0) = \frac{3}{2}$ | 3) $P'(0) = 0$ || 5) $P''(0) = -\frac{2}{3}$ raccord avec le cercle
| 2) $P(\pi) = \frac{1}{2}$ | 4) $P'(\pi) = 0$ || 6) $P''(\pi) = 0$ raccord avec la droite

Soit $P(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + g$

par V200 : $P(x) = \frac{\pi^2 - 18}{3\pi^5} x^5 + \frac{15 - \pi^2}{\pi^4} x^4 + \frac{\pi^2 - 10}{\pi^3} x^3 - \frac{1}{3} x^2 + \frac{3}{2}$

② $V_C = \pi \int_{-3/2}^{0} \left(\frac{9}{4} - x^2\right) dx + \pi \int_{0}^{\pi} P(x)^2\,dx$

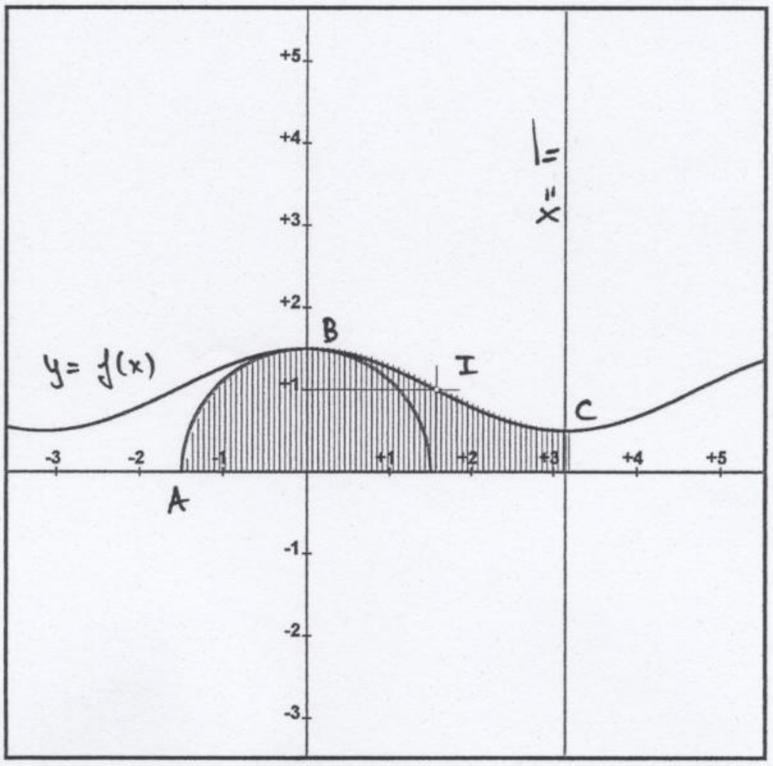
$$= \frac{\pi\,(2\pi^6 - 512\pi^3 + 47475\pi + 93555)}{41580}$$

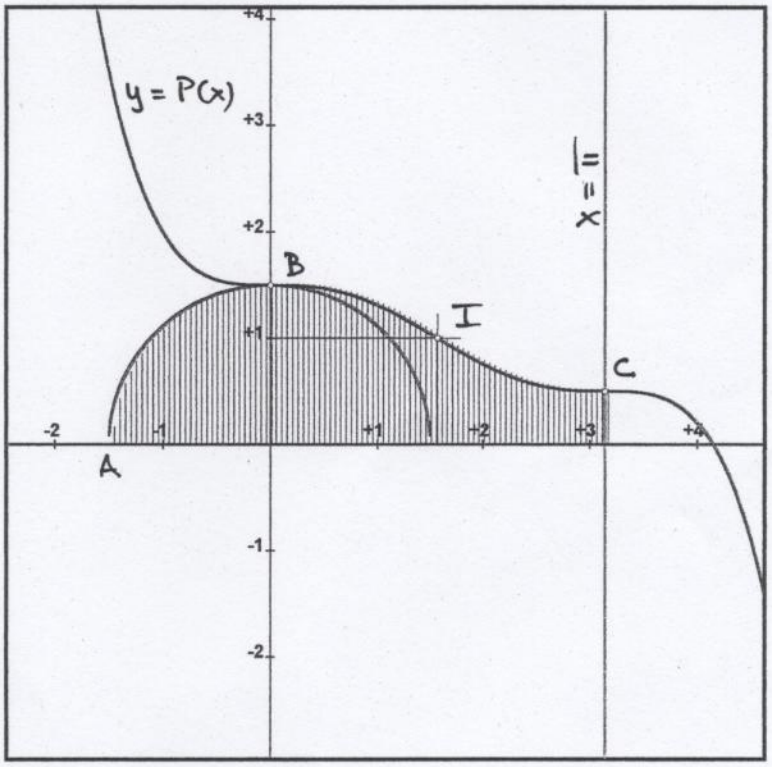
$\simeq 17,1842$ uv $\simeq 137,4$ cm³

③ Comparaison des volumes :

$V_B / V_C \simeq 1,057$ $(V_C / V_B = 0,946)$

Le volume obtenu en B) dépasse celui obtenu en C) de 5,7 % (En d'autres termes, on gagne 5,4 % en volume en remplaçant la fonction sinusoïdale par une fonction polynomiale.)


y = f(x)
A
B
I
C


y = P(x)
A
B
I
C