

Solution Question 1 (3 + 4 + 3 = 10 points)a) C.E.  $x > 0$ 

$$(\forall x \in D = ]0; +\infty[)$$

$$2^x \log_2(x) - 8 \log_2(x) - 2^{x+2} = -32$$

$$\Leftrightarrow \log_2(x)(2^x - 8) - 4(2^x - 8) = 0$$

$$\Leftrightarrow (2^x - 8)(\log_2(x) - 4) = 0$$

$$\Leftrightarrow 2^x = 8 \vee \log_2(x) = 4$$

$$\Leftrightarrow x = \boxed{3} \vee x = \boxed{16}$$

$$\in D \quad \in D$$

$$\boxed{S = \{3; 16\}}$$

b) C.E.  $x^2 - 2 \neq 0 \wedge 1 - x > 0 \Leftrightarrow x \neq -\sqrt{2} \wedge x \neq \sqrt{2} \wedge x < 1$ 

$$(\forall x \in D = ]-\infty; 1[ \setminus \{-\sqrt{2}\}) : \ln|x^2 - 2| - 2\ln(1 - x) > 0 \Leftrightarrow \ln|x^2 - 2| > \ln(x - 1)^2$$

$x$	$-\infty$	$-\sqrt{2}$	$1$
$ x^2 - 2 $	$x^2 - 2$	$2 - x^2$	

$$\boxed{1^\circ) x \in ]-\infty; -\sqrt{2}[}$$

$$\ln|x^2 - 2| > \ln(x - 1)^2 \Leftrightarrow x^2 - 2 > x^2 - 2x + 1 \Leftrightarrow x > \frac{3}{2}$$

$$S_1 = \emptyset$$

$$\boxed{2^\circ) x \in ]-\sqrt{2}; 1[}$$

$$\ln|x^2 - 2| > \ln(x - 1)^2 \Leftrightarrow 2 - x^2 > x^2 - 2x + 1 \Leftrightarrow 2x^2 - 2x - 1 < 0 \Leftrightarrow \frac{1 - \sqrt{3}}{2} < x < \frac{1 + \sqrt{3}}{2}$$

$$\approx -0,37 \quad \approx 1,37$$

$$S_2 = \left] \frac{1 - \sqrt{3}}{2}; 1[ \right.$$

$$\boxed{S = S_1 \cup S_2 = \left] \frac{1 - \sqrt{3}}{2}; 1[ \right.}$$

$$c) \lim_{x \rightarrow +\infty} \left( \frac{x-1}{x+1} \right)^{\sqrt{x}} = \lim_{x \rightarrow +\infty} e^{\frac{\sqrt{x} \ln \left( \frac{x-1}{x+1} \right)}{\rightarrow +\infty \quad \rightarrow 0}}$$

Limite de l'exposant

$$\lim_{x \rightarrow +\infty} \frac{\ln \left( \frac{x-1}{x+1} \right)}{\frac{-1}{x^2}} = \lim_{x \rightarrow +\infty} \frac{\frac{2}{(x+1)^2} \cdot \frac{x+1}{x-1}}{-\frac{1}{2} x^{-3}} = \lim_{x \rightarrow +\infty} \frac{-4x^{\frac{3}{2}}}{x^2 - 1} = \lim_{x \rightarrow +\infty} \frac{-4x^{\frac{1}{2}}}{x^{\frac{3}{2}} \left( \sqrt{x} - x^{-\frac{3}{2}} \right)} = \lim_{x \rightarrow +\infty} \frac{-4}{\sqrt{x} - x^{-\frac{3}{2}}} = 0$$

$$\text{Donc : } \lim_{x \rightarrow +\infty} \left( \frac{x-1}{x+1} \right)^{\sqrt{x}} = e^0 = \boxed{1}$$

Solution Question 2 (4 + 3 + 2 = 10 points)

$$f(x) = \begin{cases} e^{\frac{x^2}{x-1}} & \text{si } x \neq 1 \\ 0 & \text{si } x = 1 \end{cases}$$

a)  $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} e^{\frac{x^2}{x-1}} = 0$  ;  $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} e^{\frac{x^2}{x-1}} = +\infty$ .

La fonction  $f$  n'est pas continue en 1 car  $\lim_{x \rightarrow 1^+} f(x) = +\infty$ .

b)  $\lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^-} \frac{e^{\frac{x^2}{x-1}} - 0}{x - 1} = \lim_{x \rightarrow 1^-} \frac{\frac{1}{x-1}}{\frac{x^2}{e^{1-x}}} = \lim_{x \rightarrow 1^-} \frac{-\frac{1}{(x-1)^2}}{\frac{x(2-x)}{(x-1)^2} e^{\frac{x^2}{1-x}}} = \lim_{x \rightarrow 1^-} \frac{1}{x(x-2)} \cdot \frac{x^2}{e^{\frac{x^2}{x-1}}} = 0 = f'_g(0)$

$$(\forall x \in \mathbb{R} \setminus \{1\}) : f'(x) = \left( e^{\frac{x^2}{x-1}} \right)' = \frac{2x(x-1) - x^2}{(x-1)^2} e^{\frac{x^2}{x-1}} = \frac{2x^2 - 2x - x^2}{(x-1)^2} e^{\frac{x^2}{x-1}} = \frac{x(x-2)}{(x-1)^2} e^{\frac{x^2}{x-1}}$$

$f$  est dérivable à gauche en 0.

Interprétation graphique :  $G_f$  admet une demi-tangente horizontale au point  $J(0;1)$ .

b)  $\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} e^{\frac{x^2}{x-1}} = 0$

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} e^{\frac{x^2}{x-1}} = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} \frac{e^{\frac{x^2}{x-1}}}{x} = \lim_{x \rightarrow +\infty} \frac{e^{x+1+\frac{1}{x-1}}}{x} = \lim_{x \rightarrow +\infty} \frac{e^{x+1}}{x} \cdot \frac{1}{e^{\frac{1}{x-1}}} = +\infty$$

Calcul à part :  $\lim_{x \rightarrow +\infty} \frac{e^{x+1}}{x} = \lim_{x \rightarrow +\infty} \frac{e^{x+1}}{1} = +\infty$

Conclusions :

$G_f$  admet une A.V. :  $x = 1$ , une A.H.G. :  $y = 0$  et une B.P.D. dont la direction asymptotique est celle de  $(Oy)$ .

c) Tableau des variations

$x$	$-\infty$	$0$	$1$	$2$	$+\infty$
$f'(x) = \frac{x(x-2)}{(x-1)^2} e^{\frac{x^2}{x-1}}$	+	0 -		- 0 +	
$f(x) = e^{\frac{x^2}{x-1}}$	0 ↗	$\frac{1}{Max}$ ↘	0   $+\infty$	↘ $e^4_{min}$ ↗	$+\infty$

**Solution Question 3 (5 points)**

$$f(x) = (2x+1)e^x$$

$$(\forall x \in \mathbb{R}) : f'(x) = (2x+3)e^x$$

Équation de la tangente à  $G_f$  au point d'abscisse  $a$

$$\begin{aligned} t_a : y &= f'(a)(x-a) + f(a) \\ &\Leftrightarrow y = (2a+3)e^a(x-a) + (2a+1)e^a \\ &\Leftrightarrow y = ((2a+3)x - 2a^2 - a + 1)e^a \\ O(0;0) &\in t_a \end{aligned}$$

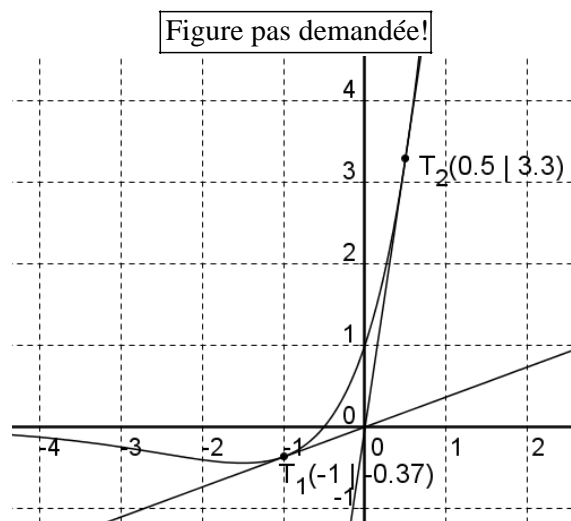
$$\Leftrightarrow 0 = ((2a+3)0 - 2a^2 - a + 1)e^a$$

$$\Leftrightarrow 2a^2 + a - 1 = 0$$

$$\Leftrightarrow a = -1 \vee a = \frac{1}{2}$$

$$t_{-1} : y = f'(-1)x \Leftrightarrow y = \frac{1}{e}x$$

$$t_{\frac{1}{2}} : y = f'\left(\frac{1}{2}\right)x \Leftrightarrow y = 4\sqrt{e}x$$



**Solution Question 4 (7+2+5=14 points)**

$$f(x) = \frac{\ln(x^3)}{x} = 3 \frac{\ln(x)}{x}$$

a)  $\text{dom } f = \text{dom } f' = \text{dom } f'' = ]0; +\infty[$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 3 \frac{\overbrace{\ln(x)}^{\rightarrow -\infty}}{\underbrace{x}_{\rightarrow 0^+}} = -\infty$$

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} 3 \frac{\overbrace{\ln(x)}^{\rightarrow +\infty}}{\underbrace{x}_{\rightarrow +\infty}} \underset{H}{=} \lim_{x \rightarrow +\infty} \frac{3}{x} = 0$$

**Conclusion :**  $G_f$  admet une A.V. :  $x = 0$  et une A.H.D. :  $y = 0$ .

$$(\forall x \in ]0; +\infty[) : f'(x) = 3 \frac{\frac{1}{x}x - \ln(x)}{x^2} = 3 \frac{1 - \ln(x)}{x^2} ; f''(x) = 3 \frac{-\frac{1}{x}x^2 + 2x \ln(x)}{(x^2)^2} = \frac{3(2 \ln(x) - 3)}{x^3}$$

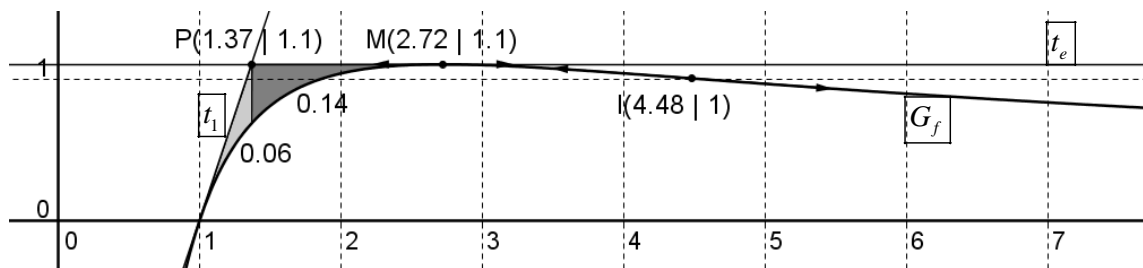
**Tableau des variations**

$x$	0	$e$	$+\infty$
$f'(x)$		+ 0 -	
$f(x)$	$-\infty$	$\nearrow \frac{3}{e}$	$\searrow 0$

**Tableau de concavité**

$x$	0	$\sqrt{e^3}$	$+\infty$
$f''(x)$		- 0 +	
$f(x)$		$\cap$ P.I. $\cup$	

Point d'inflexion :  $I\left(\sqrt{e^3}; \frac{9}{2\sqrt{e^3}}\right)$



b) Équation de la tangente à  $G_f$  au point d'abscisse  $a$

$$t_a : y = f'(a)(x-a) + f(a) \Leftrightarrow y = 3 \frac{1-\ln(a)}{a^2}(x-a) + 3 \frac{\ln(a)}{a} \Leftrightarrow y = 3 \frac{1-\ln(a)}{a^2} x + \frac{6\ln(a)-3}{a}$$

$$t_1 : y = 3x - 3$$

$$t_e : y = \frac{3}{e}$$

c)  $t_1 \cap t_e = \left\{ P\left(\frac{e+1}{e}; \frac{3}{e}\right) \right\}$

Soit  $A$  l'aire de la surface cherchée.

$$\begin{aligned} A_1 &= \int_1^{\frac{e+1}{e}} \left( 3x - 3 - 3 \frac{\ln(x)}{x} \right) dx \\ &= \left[ \frac{3}{2} x^2 - 3x - \frac{3}{2} \ln^2(x) \right]_1^{\frac{e+1}{e}} \\ &= \frac{3}{2} \left( \frac{e+1}{e} \right)^2 - 3 \frac{e+1}{e} - \frac{3}{2} \ln^2 \left( \frac{e+1}{e} \right) - \frac{3}{2} + 3 \\ &= -\frac{3}{2} \ln^2 \left( \frac{e+1}{e} \right) + \frac{3}{2e^2} \approx 0,06 u.a. \end{aligned}$$

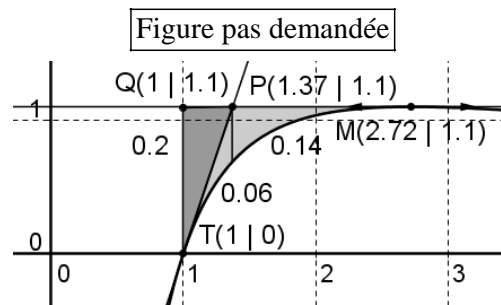
$$\begin{aligned} A_2 &= \int_{\frac{e+1}{e}}^e \left( \frac{3}{e} - 3 \frac{\ln(x)}{x} \right) dx \\ &= \left[ \frac{3}{e} x - \frac{3}{2} \ln^2(x) \right]_{\frac{e+1}{e}}^e \\ &= 3 - \frac{3}{2} - \frac{3}{e} \frac{e+1}{e} + \frac{3}{2} \ln^2 \left( \frac{e+1}{e} \right) \\ &= \frac{3}{2} \ln^2 \left( \frac{e+1}{e} \right) - \frac{3}{e} - \frac{3}{e^2} + \frac{3}{2} \approx 0,14 u.a. \end{aligned}$$

$$A = A_1 + A_2 = -\frac{3}{2} \ln^2 \left( \frac{e+1}{e} \right) + \frac{3}{2e^2} + \frac{3}{2} \ln^2 \left( \frac{e+1}{e} \right) - \frac{3}{e} - \frac{3}{e^2} + \frac{3}{2} = \boxed{\frac{3e^2-6e-3}{2e^2} u.a.} \approx 0,19 u.a.$$

OU

Soit  $Q(1; \frac{3}{e})$ , alors  $QP = \frac{1}{e}$ ,  $QT = \frac{3}{e}$  et :

$$\begin{aligned} A &= \int_1^e \left( \frac{3}{e} - 3 \frac{\ln(x)}{x} \right) dx - [TPQ] \\ &= \left[ \frac{3}{e} x - \frac{3}{2} \ln^2(x) \right]_1^e - \frac{QP \cdot QT}{2} \\ &= \frac{3}{e} e - \frac{3}{2} - \frac{3}{e} - \frac{1 \cdot \frac{3}{e}}{2} \\ &= 3 - \frac{3}{2} - \frac{3}{e} - \frac{3}{2e} - \frac{3}{2e^2} \\ &= \boxed{\frac{3e^2-6e-3}{2e^2} u.a.} \approx 0,19 u.a. \end{aligned}$$



Solution Question 5 (5 + 5 = 10 points)

$$\boxed{f(x) = \left(x - \frac{9}{2}\right) \ln(x^2 + 1)}$$

a)  $\text{dom } f = \text{dom } f' = \text{dom } f'' = \mathbb{R}$

$$(\forall x \in \mathbb{R}) : f'(x) = \ln(x^2 + 1) + \left(x - \frac{9}{2}\right) \frac{2x}{x^2 + 1} = \ln(x^2 + 1) + \frac{x(2x-9)}{x^2 + 1}$$

$$(\forall x \in \mathbb{R}) : f''(x) = \frac{2x}{x^2 + 1} + \frac{9x^2 + 4x - 9}{(x^2 + 1)^2} = \frac{2x(x^2 + 1) + 9x^2 + 4x - 9}{(x^2 + 1)^2} = \frac{2x^3 + 9x^2 + 6x - 9}{(x^2 + 1)^2}$$

$2 \cdot (-3)^3 + 9 \cdot (-3)^2 + 6 \cdot (-3) - 9$ , donc  $2x^3 + 9x^2 + 6x - 9$  est divisible par  $x + 3$ .

Schéma de Horner

	2	9	6	-9
↓ +	0	-6	-9	9
↗ · (-3)	2	3	-3	0

$$2x^3 + 9x^2 + 6x - 9 = (x+3)(2x^2 + 3x - 3) = 2(x+3)\left(x - \frac{-3-\sqrt{33}}{4}\right)\left(x - \frac{-3+\sqrt{33}}{4}\right)$$

$$f''(x) = 0 \Leftrightarrow x = -3 \vee x = \frac{-3-\sqrt{33}}{4} \vee x = \frac{-3+\sqrt{33}}{4}$$

Tableau de concavité

$x$	$-\infty$	$-3$	$\frac{-3-\sqrt{33}}{4}$	$\frac{-3+\sqrt{33}}{4}$	$+\infty$
$f''(x)$	$-$	$0$	$+$	$0$	$-$
$G_f$	$\cap$	$P.I.$	$\cup$	$P.I.$	$\cap$

$G_f$  admet les trois points d'inflexion  $I_1, I_2$  et  $I_3$  avec :

$$x_{I_1} = -3 ; x_{I_2} = \frac{-3-\sqrt{33}}{4} \text{ et } ; x_{I_3} = \frac{-3+\sqrt{33}}{4}$$

b)  $(\forall x \in \mathbb{R}) : \ln(x^2 + 1) \geq 0$  et  $\ln(x^2 + 1) = 0 \Leftrightarrow x^2 + 1 = 1 \Leftrightarrow x = 0$

Tableau des signes

$x$	$-\infty$	$0$	$\frac{9}{2}$	$+\infty$
$f(x) = (x - \frac{9}{2})\ln(x^2 + 1)$	$-$	$0$	$-$	$+$

A

$$= -\int_0^{\frac{9}{2}} (x - \frac{9}{2})\ln(x^2 + 1) dx$$

Intégration par parties

$$\begin{array}{l} u(x) = \ln(x^2 + 1) \quad | \quad v(x) = \frac{1}{2}x^2 - \frac{9}{2}x \\ u'(x) = \frac{2x}{x^2+1} \quad | \quad v'(x) = x - \frac{9}{2} \end{array}$$

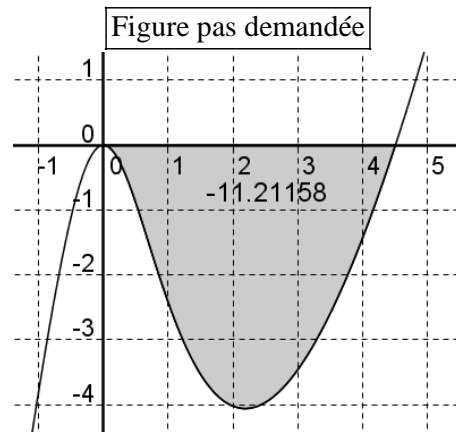
$$= -\left[ \left( \frac{1}{2}x^2 - \frac{9}{2}x \right) \ln(x^2 + 1) \right]_0^{\frac{9}{2}} + \int_0^{\frac{9}{2}} \frac{x^3 - 9x^2}{x^2 + 1} dx$$

$$= \frac{81}{8} \ln\left(\frac{85}{4}\right) + \int_0^{\frac{9}{2}} \left( -\frac{x}{x^2+1} + \frac{9}{x^2+1} + x - 9 \right) dx$$

$$= \frac{81}{8} \ln\left(\frac{85}{4}\right) + \left[ -\frac{1}{2} \ln(x^2 + 1) + 9 \arctan(x) + \frac{1}{2}x^2 - 9x \right]_0^{\frac{9}{2}}$$

$$= \frac{81}{8} \ln\left(\frac{85}{4}\right) - \frac{1}{2} \ln\left(\frac{85}{4}\right) + 9 \arctan\left(\frac{9}{2}\right) - \frac{243}{8}$$

$$= \boxed{\frac{77}{8} \ln\left(\frac{85}{4}\right) + 9 \arctan\left(\frac{9}{2}\right) - \frac{243}{8} \text{ u.a.}} \approx 11,21 \text{ u.a.}$$



Solution Question 6 (7 points)

$$c' : x^2 + \left(y + \frac{1}{2}\right)^2 = \left(\frac{\sqrt{5}}{2}\right)^2 \Leftrightarrow y + \frac{1}{2} = \pm \sqrt{\frac{5}{4} - x^2} \Leftrightarrow y = \frac{1}{2} \sqrt{5 - 4x^2} - \frac{1}{2} \vee y = -\frac{1}{2} \sqrt{5 - 4x^2} - \frac{1}{2}$$

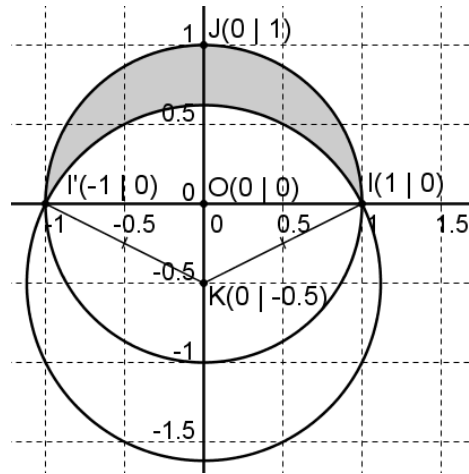
$c \cap c'$

$$\begin{cases} x^2 + y^2 = 1 \\ x^2 + y^2 + y - 1 = 0 \end{cases} \Leftrightarrow (x; y) = (-1; 0) \vee (x; y) = (1; 0)$$

$$c \cap c' = \{I'(-1; 0); I'(1; 0)\}$$

V

$$\begin{aligned}
 &= \underbrace{\frac{4}{3}\pi \cdot 1^3}_{\text{volume d'une boule}} - \pi \int_{-1}^1 \left( \frac{1}{2}\sqrt{5-4x^2} - \frac{1}{2} \right)^2 dx \\
 &= \frac{4}{3}\pi - \pi \int_{-1}^1 \left( -x^2 + 1 + \frac{1}{2} - \frac{1}{2}\sqrt{5-4x^2} \right) dx \\
 &= \frac{4}{3}\pi - \pi \left[ x - \frac{1}{3}x^3 \right]_{-1}^1 + \pi \int_{-1}^1 \left( \frac{1}{2}\sqrt{5-4x^2} - \frac{1}{2} \right) dx \\
 &= \cancel{\frac{4}{3}\pi} - \cancel{\frac{4}{3}\pi} + \pi \left\{ \begin{array}{l} \text{secteur circulaire} \\ \text{de centre } K \text{ de} \\ \text{rayon } \frac{\sqrt{5}}{2} \text{ et d'angle } \widehat{IKI'} \end{array} \right\} - [KII'] \\
 &= \pi \frac{\left(\frac{\sqrt{5}}{2}\right)^2 \cdot 2 \cdot \arctan(2)}{2} - \pi \frac{2 \cdot \frac{1}{2}}{2} \\
 &= \boxed{\frac{5}{4}\pi \arctan(2) - \frac{\pi}{2} u.v.} \approx 2,78 u.v.
 \end{aligned}$$



OU

V

$$\begin{aligned}
 &= \underbrace{\frac{4}{3}\pi \cdot 1^3}_{\text{volume d'une boule}} - \pi \int_{-1}^1 \left( \frac{1}{2}\sqrt{5-4x^2} - \frac{1}{2} \right)^2 dx \\
 &= \frac{4}{3}\pi - \pi \int_{-1}^1 \left( -x^2 + \frac{3}{2} - \frac{1}{2}\sqrt{5-4x^2} \right) dx \\
 &= \frac{4}{3}\pi - \pi \left[ \frac{3}{2}x - \frac{1}{3}x^3 \right]_{-1}^1 + \frac{\pi}{2} \int_{-1}^1 \sqrt{5-4x^2} dx \\
 &= \frac{4}{3}\pi - \frac{7}{3}\pi + \frac{\sqrt{5}\pi}{2} \int_{-1}^1 \sqrt{1 - \left(\frac{2\sqrt{5}}{5}x\right)^2} dx
 \end{aligned}$$

Changement de variable

$$\begin{aligned}
 &\frac{2\sqrt{5}}{5}x = \sin(u) ; dx = \frac{\sqrt{5}}{2} \cos(u) du ; x = -1 \Rightarrow u = \arcsin\left(-\frac{2\sqrt{5}}{5}\right) ; x = 1 \Rightarrow u = \arcsin\left(\frac{2\sqrt{5}}{5}\right) \\
 &= -\pi + \frac{\sqrt{5}\pi}{2} \int_{\arcsin\left(-\frac{2\sqrt{5}}{5}\right)}^{\arcsin\left(\frac{2\sqrt{5}}{5}\right)} \frac{\sqrt{5}}{2} \cos^2(u) du \\
 &= -\pi + \frac{5\pi}{4} \int_{\arcsin\left(-\frac{2\sqrt{5}}{5}\right)}^{\arcsin\left(\frac{2\sqrt{5}}{5}\right)} \left( \frac{1}{2} + \frac{1}{2} \cos(2u) \right) du \\
 &= -\pi + \frac{5\pi}{4} \left[ \frac{1}{2}u + \frac{1}{4} \sin(2u) \right]_{\arcsin\left(-\frac{2\sqrt{5}}{5}\right)}^{\arcsin\left(\frac{2\sqrt{5}}{5}\right)} \\
 &= -\pi + \frac{5\pi}{4} \left[ \begin{array}{l} \frac{1}{2} \arcsin\left(\frac{2\sqrt{5}}{5}\right) + \frac{1}{4} \sin\left(\arcsin\left(\frac{2\sqrt{5}}{5}\right)\right) \cos\left(\arcsin\left(\frac{2\sqrt{5}}{5}\right)\right) \\ -\frac{1}{2} \arcsin\left(-\frac{2\sqrt{5}}{5}\right) - \frac{1}{4} \sin\left(\arcsin\left(-\frac{2\sqrt{5}}{5}\right)\right) \cos\left(\arcsin\left(-\frac{2\sqrt{5}}{5}\right)\right) \end{array} \right] \\
 &= -\pi + \frac{5\pi}{4} \left[ \arcsin\left(\frac{2\sqrt{5}}{5}\right) + \frac{2\sqrt{5}}{5} \sqrt{1 - \left(\frac{2\sqrt{5}}{5}\right)^2} \right] \\
 &= \boxed{\frac{5}{4}\pi \arcsin\left(\frac{2\sqrt{5}}{5}\right) - \frac{\pi}{2} u.v.} \approx 2,78 u.v.
 \end{aligned}$$

Solution Question 7 (5 points)

$$a) \int \frac{\sin^2(2x)}{\cos(2x)} dx = \int \frac{1 - \cos^2(2x)}{\cos(2x)} dx = \int \frac{1}{\cos(2x)} dx - \int \cos(2x) dx = \int \frac{1}{\cos(2x)} dx - \frac{1}{2} \sin(2x)$$

Calcul de  $\int \frac{1}{\cos(2x)} dx$

Changement de variable

$$u = \tan(x) ; dx = \frac{du}{1+u^2} ; \cos(2x) = \frac{1-u^2}{1+u^2}$$

$$\int \frac{1}{\cos(2x)} dx = \int \frac{\frac{du}{1+u^2}}{\frac{1-u^2}{1+u^2}} = \int \frac{du}{1-u^2} = \int \left( \frac{\frac{1}{2}}{1-u} + \frac{\frac{1}{2}}{1+u} \right) du = \frac{1}{2} \ln \left| \frac{1+u}{1-u} \right| + c = \frac{1}{2} \ln \left| \frac{1+\tan(x)}{1-\tan(x)} \right| + c \quad (c \in \mathbb{R})$$

$$(\forall \lambda \in [0; \frac{\pi}{4}[) : I(\lambda) = \left[ \frac{1}{2} \ln \left| \frac{1+\tan(x)}{1-\tan(x)} \right| - \frac{1}{2} \sin(2x) \right]_0^\lambda = \frac{1}{2} \ln \left| \frac{1+\tan(\lambda)}{1-\tan(\lambda)} \right| - \frac{1}{2} \sin(2\lambda)$$

$$b) \lim_{\lambda \rightarrow \frac{\pi}{4}^-} I(\lambda) = \lim_{\lambda \rightarrow \frac{\pi}{4}^-} \left( \frac{1}{2} \ln \left( \frac{\overbrace{1+\tan(\lambda)}^{\rightarrow 2}}{\underbrace{1-\tan(\lambda)}_{\rightarrow 0^+}} \right) - \frac{1}{2} \underbrace{\sin(2\lambda)}_{\rightarrow 1} \right) = +\infty$$