

Corrigé C&D_MATH2_QE...

Question 1 (4+(6+4)=14 points)

(1) Question de cours, voir livre EM66 page 67

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$$(2) (a) \quad \log_{\frac{1}{2}}(2-x) - \log_{\sqrt{2}} \sqrt{x+4} \geq \log_2 \frac{1}{(x+3)^2} \quad (I)$$

C.E. : 1) $2-x > 0 \Leftrightarrow x < 2$

2) $x+4 > 0 \Leftrightarrow x > -4$

3) $(x+3)^2 > 0 \Leftrightarrow (x+3)^2 \neq 0 \Leftrightarrow x \neq -3$

$$D =]-4; -3[\cup]-3; 2[$$

$$(\forall x \in D) \quad (I) \Leftrightarrow \frac{\log_2(2-x)}{\log_2\left(\frac{1}{2}\right)} - \frac{\log_2 \sqrt{x+4}}{\log_2(\sqrt{2})} \geq \log_2(x+3)^{-2}$$

$$\Leftrightarrow -\log_2(2-x) - 2\log_2 \sqrt{x+4} \geq -\log_2(x+3)^2$$

$$\Leftrightarrow \log_2(2-x) + \log_2(x+4) \leq \log_2(x+3)^2$$

$$\Leftrightarrow \log_2(2-x)(x+4) \leq \log_2(x+3)^2$$

$$\stackrel{bij \nearrow}{\Leftrightarrow} (2-x)(x+4) \leq (x+3)^2$$

$$\Leftrightarrow 2x^2 + 8x + 1 \geq 0$$

Posons : $2x^2 + 8x + 1 = 0$

$\Delta = 56$

$$x = \frac{-8 \pm 2\sqrt{14}}{4} = \left\langle \begin{array}{l} \frac{-4 + \sqrt{14}}{2} \approx -0,13 \\ \frac{-4 - \sqrt{14}}{2} \approx -3,87 \end{array} \right.$$

x	-4	$\frac{-4 - \sqrt{14}}{2}$	-3	$\frac{-4 + \sqrt{14}}{2}$	2
$2x^2 + 8x + 1$	+	+	0	-	-
	+	+	-	-	+

$$S_{\mathbb{R}} = \left] -4; \frac{-4 - \sqrt{14}}{2} \right] \cup \left[\frac{-4 + \sqrt{14}}{2}; 2 \right[$$

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$$(b) \quad 9 + 10 \cdot 5^{-1-x} = 5^{x+1} \quad D = \mathbb{R}$$

$$\Leftrightarrow 9 + 10 \cdot \frac{1}{5} \cdot 5^{-x} = 5 \cdot 5^x \quad | \cdot 5^x \neq 0$$

$$\Leftrightarrow 9 \cdot 5^x + 2 - 5 \cdot 5^{2x} = 0$$

$$\Leftrightarrow 5 \cdot 5^{2x} - 9 \cdot 5^x - 2 = 0$$

Posons : $5^x = u$, donc $u > 0$

L'équation s'écrit :

$$5u^2 - 9u - 2 = 0 \quad \Delta = 121 \quad u = \frac{9 \pm 11}{10} = \left\langle \begin{array}{l} 2 \\ -\frac{1}{5} \end{array} \right. \text{ à écarter}$$

$$u = 2 \Leftrightarrow 5^x = 2$$

$$\Leftrightarrow 5^x = 5^{\log_5 2}$$

$\text{bij } \nearrow$

$$\Leftrightarrow x = \log_5 2$$

$$S_{\mathbb{R}} = \{\log_5 2\}$$

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Question 2 (3+(4+4)+4+(3+3)=21 points)

$$(1) \quad \lim_{x \rightarrow 0} (1-2x)^{\frac{1}{2x}} \quad (\text{f.i. "1}^\infty \text{"}) = \lim_{x \rightarrow 0} e^{\frac{1}{2x} \ln(1-2x)} = e^{-1} = \frac{1}{e}$$

$$\text{Car : } \lim_{x \rightarrow 0} \frac{1}{2x} \ln(1-2x) = \lim_{x \rightarrow 0} \frac{\ln(1-2x)}{2x} \rightarrow 0$$

$$\stackrel{[H]}{=} \lim_{x \rightarrow 0} \frac{-2}{2(1-2x)}$$

$$= \lim_{x \rightarrow 0} \frac{-1}{1-2x}$$

$$= -1$$

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$$(2) (a) \quad \int_0^3 \frac{3x+1}{\sqrt{9-x^2}} dx = \int_0^3 \left(\frac{3x}{\sqrt{9-x^2}} + \frac{1}{3 \cdot \sqrt{1-\left(\frac{x}{3}\right)^2}} \right) dx$$

$$= -\frac{3}{2} \int_0^3 \frac{-2x}{\sqrt{9-x^2}} dx + \int_0^3 \frac{\frac{1}{3}}{\sqrt{1-\left(\frac{x}{3}\right)^2}} dx$$

$$\begin{aligned}
&= -\frac{3}{2} \left[2\sqrt{9-x^2} \right]_0^3 + \left[\text{Arcsin} \left(\frac{x}{3} \right) \right]_0^3 \\
&= -\frac{3}{2} (0-6) + \text{Arcsin} 1 - \text{Arcsin} 0 \\
&= 9 + \frac{\pi}{2}
\end{aligned}$$

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(b) $\int_2^{2e} (1+x^2) \ln\left(\frac{x}{2}\right) dx$ i.p.p

Posons : $u(x) = \ln\left(\frac{x}{2}\right)$ $v'(x) = 1+x^2$

$u'(x) = \frac{1}{x}$ $v(x) = x + \frac{1}{3}x^3$

$$\begin{aligned}
&= \left[\left(x + \frac{1}{3}x^3 \right) \ln\left(\frac{x}{2}\right) \right]_2^{2e} - \int_2^{2e} \frac{1}{x} \cdot \left(x + \frac{1}{3}x^3 \right) dx \\
&= \left[\left(x + \frac{1}{3}x^3 \right) \ln\left(\frac{x}{2}\right) \right]_2^{2e} - \int_2^{2e} \left(1 + \frac{1}{3}x^2 \right) dx \\
&= \left[\left(x + \frac{1}{3}x^3 \right) \ln\left(\frac{x}{2}\right) - x - \frac{1}{9}x^3 \right]_2^{2e} \\
&= \left(2e + \frac{8}{3}e^3 \right) \ln e - 2e - \frac{8}{9}e^3 - \left(2 + \frac{8}{3} \right) \ln 1 + 2 + \frac{8}{9} \\
&= \frac{26}{9} + \frac{16}{9}e^3
\end{aligned}$$

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(3) $f(x) = \sin(2x) \cdot \cos^2 x$

$$\begin{aligned}
F(x) &= \int \sin(2x) \cdot \cos^2 x dx \\
&= \int 2 \sin x \cdot \cos^3 x dx \\
&= -\frac{1}{2} \cos^4 x + c, \quad c \in \mathbb{R}
\end{aligned}$$

$$F\left(\frac{\pi}{4}\right) = \frac{1}{8} \Leftrightarrow -\frac{1}{2} \cos^4\left(\frac{\pi}{4}\right) + c = \frac{1}{8} \Leftrightarrow -\frac{1}{2} \cdot \left(\frac{\sqrt{2}}{2}\right)^4 + c = \frac{1}{8} \Leftrightarrow c = \frac{1}{4}$$

La primitive à déterminer est : $F_{\frac{1}{4}} : x \mapsto -\frac{1}{2} \cos^4 x + \frac{1}{4}$

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$$(4) \text{ (a)} \quad f(x) = \frac{1}{4}x^2 \quad \text{dom } f = \mathbb{R} \quad (\forall x \in \mathbb{R}) \quad f(x) \geq 0$$

$$g(x) = 2\sqrt{x} \quad \text{dom } g = \mathbb{R}_+ \quad (\forall x \in \mathbb{R}_+) \quad g(x) \geq 0$$

$$(\forall x \in \mathbb{R}_+) \quad f(x) \geq g(x)$$

$$\Leftrightarrow \frac{1}{4}x^2 \geq 2\sqrt{x}$$

$$\Leftrightarrow \begin{matrix} x^2 & \geq & 8\sqrt{x} \\ \geq 0 & & \geq 0 \end{matrix} \quad | \text{ élever au carré}$$

$$\Leftrightarrow x^4 \geq 64x$$

$$\Leftrightarrow x \cdot (x^3 - 64) \geq 0$$

$$\Leftrightarrow x \cdot (x-4) \cdot \underbrace{(x^2 + 4x + 16)}_{> 0, \text{ car } \Delta = -48 < 0} \geq 0$$

$$\Leftrightarrow x \cdot (x-4) \geq 0$$

x	0	4	$+\infty$
$f(x) - g(x)$	0	-	0
		+	
		G_g/G_f	
		G_f/G_g	

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$$(b) \quad \text{On a : } (\forall x \in [0; 4]) \quad g(x) \geq f(x) \geq 0$$

$$V = \pi \cdot \int_0^4 \left([g(x)]^2 - [f(x)]^2 \right) dx$$

$$= \pi \cdot \int_0^4 \left([2\sqrt{x}]^2 - \left[\frac{1}{4}x^2 \right]^2 \right) dx$$

$$= \pi \cdot \int_0^4 \left(4x - \frac{1}{16}x^4 \right) dx$$

$$= \pi \cdot \left[2x^2 - \frac{1}{80}x^5 \right]_0^4$$

$$= \pi \cdot \left(32 - \frac{1}{80} \cdot 1024 \right)$$

$$= \frac{96}{5} \pi \quad \text{u.v.} \quad (\approx 60,32 \text{ u.v.})$$

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Question 3 (13+4+8=25 points)

$$f(x) = (2x+1)^2 \cdot e^{-x}$$

(1) $dom f = \mathbb{R} = dom_c f$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \underbrace{(2x+1)^2}_{\rightarrow +\infty} \cdot \underbrace{e^{-x}}_{\rightarrow +\infty} = +\infty \quad \text{donc pas d'AHG}$$

$$\lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} \underbrace{x}_{\rightarrow -\infty} \cdot \underbrace{\left(2 + \frac{1}{x}\right)^2}_{\rightarrow 4} \cdot \underbrace{e^{-x}}_{\rightarrow +\infty} = -\infty \quad \text{donc pas d'AOG}$$

mais BP dans la direction de (Oy)

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \underbrace{(2x+1)^2}_{\rightarrow +\infty} \cdot \underbrace{e^{-x}}_{\rightarrow 0} = \lim_{x \rightarrow +\infty} \frac{\boxed{(2x+1)^2} \rightarrow +\infty}{\boxed{e^x} \rightarrow +\infty} \stackrel{[H]}{=} \lim_{x \rightarrow +\infty} \frac{\boxed{4(2x+1)} \rightarrow +\infty}{\boxed{e^x} \rightarrow +\infty}$$

$$\stackrel{[H]}{=} \lim_{x \rightarrow +\infty} \frac{8}{e^x} = 0$$

donc AHD : $y = 0$

$$dom f' = \mathbb{R}$$

$$\begin{aligned} f'(x) &= 4(2x+1) \cdot e^{-x} - (2x+1)^2 e^{-x} \\ &= (2x+1) \cdot (3-2x) \cdot e^{-x} \\ &= (-4x^2 + 4x + 3) \cdot e^{-x} \end{aligned}$$

$$f'(x) = 0 \Leftrightarrow (2x+1) \cdot (3-2x) \cdot \underbrace{e^{-x}}_{>0} = 0 \Leftrightarrow x = -\frac{1}{2} \vee x = \frac{3}{2}$$

$$dom f'' = \mathbb{R}$$

$$f''(x) = (-8x+4) \cdot e^{-x} - (-4x^2 + 4x + 3) \cdot e^{-x} = (4x^2 - 12x + 1) \cdot e^{-x}$$

$$f''(x) = 0 \Leftrightarrow (4x^2 - 12x + 1) \cdot \underbrace{e^{-x}}_{>0} = 0 \Leftrightarrow 4x^2 - 12x + 1 = 0$$

$$\Delta = 128$$

$$x = \frac{12 \pm 8\sqrt{2}}{8} = \left\langle \begin{array}{l} \frac{3+2\sqrt{2}}{2} \approx 2,9 \\ \frac{3-2\sqrt{2}}{2} \approx 0,1 \end{array} \right.$$

Tableau récapitulatif

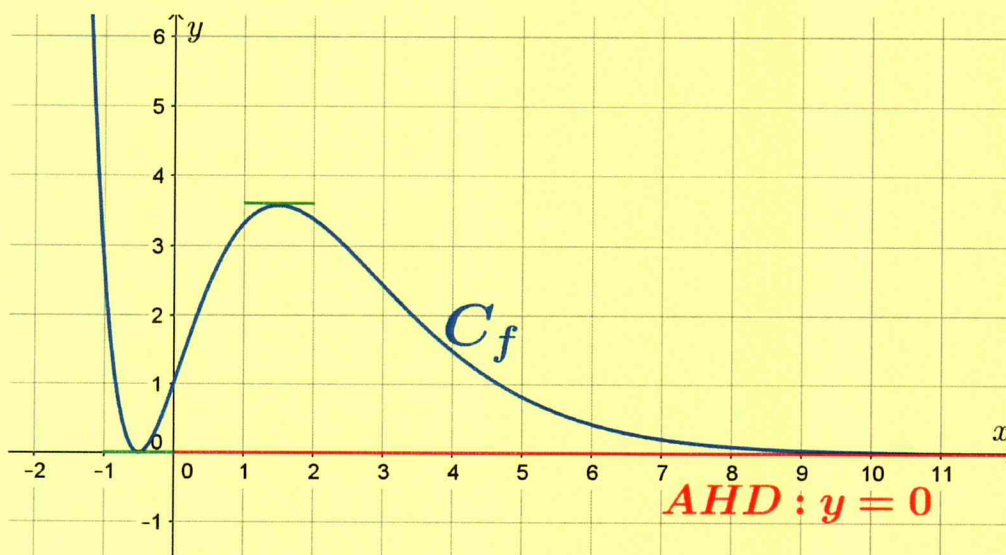
x	$-\infty$	$-\frac{1}{2}$	$\frac{3-2\sqrt{2}}{2}$	$\frac{3}{2}$	$\frac{3+2\sqrt{2}}{2}$	$+\infty$
$f'(x)$		- 0 +		+ 0 -		
$f''(x)$		+ + 0 -		- 0 +		
f	$+\infty$	\searrow 0 \nearrow min	\nearrow $\approx 1,3$	\nearrow $\frac{16}{e\sqrt{e}} \approx 3,6$ Max	\searrow $\approx 2,5$	\searrow 0
C_f	$BPdir(Oy)$	PI		PI		AHD: $y=0$

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(2) Tableau de valeurs

x	-1	$-\frac{1}{2}$	0	1	$\frac{3}{2}$	2	3	4	5
$f(x)$	2,7	0	1	3,3	3,6	3,4	2,4	1,5	0,8

Représentation graphique



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(3) (a) $(\forall x \in \mathbb{R}) \quad f(x) \geq 0$

$$\text{Donc } A(\lambda) = \int_{-\frac{1}{2}}^{\lambda} f(x) dx$$

$$\text{Soit } I = \int f(x) dx = \int (2x+1)^2 \cdot e^{-x} dx \quad \text{ipp}$$

$$u(x) = (2x+1)^2$$

$$v'(x) = e^{-x}$$

$$u'(x) = 4 \cdot (2x+1)$$

$$v(x) = -e^{-x}$$

$$I = -(2x+1)^2 \cdot e^{-x} + 4 \int (2x+1) \cdot e^{-x} dx \quad \text{ipp}$$

$$u(x) = 2x+1$$

$$v'(x) = e^{-x}$$

$$u'(x) = 2$$

$$v(x) = -e^{-x}$$

$$I = -(2x+1)^2 \cdot e^{-x} - 4 \cdot (2x+1) \cdot e^{-x} + 8 \int e^{-x} dx$$

$$= -(2x+1)^2 \cdot e^{-x} - 4 \cdot (2x+1) \cdot e^{-x} - 8 \cdot e^{-x} + c$$

$$= (-4x^2 - 12x - 13) \cdot e^{-x} + c, \quad c \in \mathbb{R}$$

$$A(\lambda) = \left[(-4x^2 - 12x - 13) \cdot e^{-x} \right]_{-\frac{1}{2}}^{\lambda} = (-4\lambda^2 - 12\lambda - 13) \cdot e^{-\lambda} + (1 - 6 + 3) \cdot e^{\frac{1}{2}}$$

$$= (-4\lambda^2 - 12\lambda - 13) \cdot e^{-\lambda} + 8\sqrt{e}$$

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$$(b) \quad \lim_{\lambda \rightarrow +\infty} A(\lambda) = \lim_{\lambda \rightarrow +\infty} \left[(-4\lambda^2 - 12\lambda - 13) \cdot e^{-\lambda} + 8\sqrt{e} \right]$$

$$\text{or } \lim_{\lambda \rightarrow +\infty} \underbrace{(-4\lambda^2 - 12\lambda - 13)}_{\rightarrow -\infty} \cdot \underbrace{e^{-\lambda}}_{\rightarrow 0} = \lim_{\lambda \rightarrow -\infty} \frac{\boxed{-4\lambda^2 - 12\lambda - 13} \rightarrow -\infty}{\boxed{e^{\lambda}} \rightarrow +\infty}$$

$$\stackrel{[H]}{=} \lim_{\lambda \rightarrow +\infty} \frac{\boxed{-8\lambda - 12} \rightarrow -\infty}{\boxed{e^{\lambda}} \rightarrow +\infty}$$

$$\stackrel{[H]}{=} \lim_{\lambda \rightarrow +\infty} \frac{-8}{e^{\lambda}}$$

$$= 0$$

$$\text{donc } \boxed{A(\lambda) = 8\sqrt{e} \text{ u.a. } (cm^2)}$$

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