



DISCIPLINE	SECTION(S)	ÉPREUVE ÉCRITE	
Mathématiques 2	CC	Date de l'épreuve :	13.06.22
		Durée de l'épreuve :	08:15 - 11:10

Question 1

[4 points]

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Question 2

[5+4+3=12 points]

$$1) \log_{\sqrt{3}}(1+2x) + \log_{\frac{1}{3}}(1-x) \leq \log_3(1-2x) \quad (I)$$

Conditions d'existence :

- $1+2x > 0 \iff x > -\frac{1}{2}$
- $1-x > 0 \iff x < 1$
- $1-2x > 0 \iff x < \frac{1}{2}$

$$D = ]-\frac{1}{2}; \frac{1}{2}[$$

$$(\forall x \in D) \quad (I) \iff \frac{\log_3(1+2x)}{\log_3(\sqrt{3})} + \frac{\log_3(1-x)}{\log_3(\frac{1}{3})} \leq \log_3(1-2x)$$

$$\iff \frac{\log_3(1+2x)}{\frac{1}{2}\log_3 3} + \frac{\log_3(1-x)}{-\log_3 3} \leq \log_3(1-2x)$$

$$\iff 2\log_3(1+2x) - \log_3(1-x) \leq \log_3(1-2x)$$

$$\iff \log_3(1+2x)^2 \leq \log_3(1-2x) + \log_3(1-x)$$

$$\iff \log_3(1+2x)^2 \leq \log_3[(1-2x)(1-x)]$$

$$\iff (1+2x)^2 \leq (1-2x)(1-x) \quad \text{car } \log_3 \text{ est une bij. str. croissante sur } \mathbb{R}_+^*$$

$$\iff 4x^2 + 4x + 1 \leq 1 - 3x + 2x^2$$

$$\iff 2x^2 + 7x \leq 0$$

$$\iff x(2x+7) \leq 0$$

$$\iff x \in [-\frac{7}{2}; 0]$$

$$S = [-\frac{7}{2}; 0] \cap D = ]-\frac{1}{2}; 0]$$

[5]

$$2) \frac{-1 - e^{-2x}}{1 + e^{2x}} + \frac{1}{4} = 0 \quad (E)$$

Condition d'existence :  $1 + e^{2x} \neq 0$  toujours vérifié, donc  $D = \mathbb{R}$ .

$$(\forall x \in \mathbb{R}) \quad (E) \iff 4 \cdot (-1 - e^{-2x}) + 1 + e^{2x} = 0$$

$$\iff e^{2x} - 3 - 4e^{-2x} = 0 \quad | \cdot e^{2x} \neq 0$$

$$\iff e^{4x} - 3e^{2x} - 4 = 0$$

Posons  $e^{2x} = y$ , avec  $y > 0$ .

$$(\forall y \in \mathbb{R}_{+}^*) \quad (E) \iff y^2 - 3y - 4 = 0 \iff \underbrace{y = -1}_{\text{à écarter}} \vee y = 4$$

$$\begin{aligned}
 y = 4 &\iff e^{2x} = 4 \\
 &\iff 2x = \ln 4 \\
 &\iff x = \frac{1}{2} \ln 4 \\
 &\iff x = \ln 2
 \end{aligned}$$

$$S = \{\ln 2\} \quad [4]$$

$$\begin{aligned}
 3) \lim_{x \rightarrow -\infty} \left( \frac{x-1}{x+5} \right)^{\frac{1}{2}x+1} &= \lim_{x \rightarrow -\infty} \left( \frac{x+5-6}{x+5} \right)^{\frac{1}{2}x+1} \\
 &= \lim_{x \rightarrow -\infty} \left( 1 + \frac{-6}{x+5} \right)^{\frac{1}{2}x+1} \\
 &= \lim_{y \rightarrow 0} (1+y)^{\frac{1}{2} \cdot \left( \frac{-6}{y} - 5 \right) + 1} \quad \left( y = \frac{-6}{x+5} \iff x = -\frac{6}{y} - 5; \text{ si } x \rightarrow -\infty, \text{ alors } y \rightarrow 0 \right) \\
 &= \lim_{y \rightarrow 0} (1+y)^{-\frac{3}{y} - \frac{3}{2}} \\
 &= \lim_{y \rightarrow 0} \left[ \underbrace{\left( (1+y)^{\frac{1}{y}} \right)}_{\rightarrow e} \right]^{-3} \cdot \underbrace{(1+y)^{-\frac{3}{2}}}_{\rightarrow 1} \\
 &= e^{-3} \\
 &= \frac{1}{e^3}
 \end{aligned}$$

[3]

Question 3

[(4+1,5+5,5+1+3)+5=20 points]

$$f(x) = \frac{1}{2}(x^2 - 6x + 10)e^{x-1}$$

1) (a)  $\text{Dom } f = \mathbb{R}$ . [0.5]

$$\begin{aligned}
 \lim_{x \rightarrow -\infty} f(x) &= \lim_{x \rightarrow -\infty} \frac{1}{2} \cdot \underbrace{(x^2 - 6x + 10)}_{\rightarrow +\infty} \cdot \underbrace{e^{x-1}}_{\rightarrow 0} \\
 &= \lim_{x \rightarrow -\infty} \frac{1}{2} \cdot \frac{x^2 - 6x + 10}{e^{-x+1}} \quad \frac{\infty}{\infty} \\
 &\stackrel{H}{=} \lim_{x \rightarrow -\infty} \frac{1}{2} \cdot \frac{2x - 6}{-e^{-x+1}} \quad \frac{\infty}{\infty} \\
 &\stackrel{H}{=} \lim_{x \rightarrow -\infty} \frac{1}{2} \cdot \frac{2}{e^{-x+1}} \\
 &= 0
 \end{aligned}$$

A.H.G :  $y = 0$

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{1}{2} \cdot \underbrace{(x^2 - 6x + 10)}_{\rightarrow +\infty} \cdot \underbrace{e^{x-1}}_{\rightarrow +\infty} = +\infty \quad \text{pas d'A.H.D.} \quad [1.5]$$

$$\text{Calcul à part : } \lim_{x \rightarrow +\infty} (x^2 - 6x + 10) = \lim_{x \rightarrow +\infty} x^2 \left( 1 - \frac{6}{x} + \frac{10}{x^2} \right) = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} \frac{1}{2} \underbrace{\left(x - 6 + \frac{10}{x}\right)}_{\rightarrow +\infty} \cdot \underbrace{e^{x-1}}_{\rightarrow +\infty} = +\infty$$

pas d'A.O.D. (B.P. dir. (Oy)) [2]

(b)  $\text{Dom} f' = \mathbb{R}$

$$\begin{aligned} f'(x) &= \frac{1}{2} \cdot (2x - 6) \cdot e^{x-1} + \frac{1}{2} \cdot (x^2 - 6x + 10) \cdot e^{x-1} \\ &= \frac{1}{2} \cdot (x^2 - 6x + 10 + 2x - 6) \cdot e^{x-1} \\ &= \frac{1}{2} \cdot (x^2 - 4x + 4) \cdot e^{x-1} \\ &= \frac{(x - 2)^2 \cdot e^{x-1}}{2} \end{aligned}$$

[1,5]

(c)  $\text{Dom} f'' = \mathbb{R}$

$$\begin{aligned} f''(x) &= \frac{1}{2} \cdot 2 \cdot (x - 2) \cdot e^{x-1} + \frac{(x - 2)^2}{2} \cdot e^{x-1} \\ &= \frac{1}{2} \cdot (x - 2)(2 + x - 2) \cdot e^{x-1} \\ &= \frac{x(x - 2) \cdot e^{x-1}}{2} \end{aligned}$$

[1]

$$\begin{aligned} f'(x) = 0 &\iff \frac{(x - 2)^2 \cdot e^{x-1}}{2} = 0 \\ &\iff (x - 2)^2 = 0 \\ &\iff x = 2 \end{aligned}$$

[1]

$$\begin{aligned} f''(x) = 0 &\iff \frac{x(x - 2) \cdot e^{x-1}}{2} = 0 \\ &\iff x(x - 2) = 0 \\ &\iff x = 0 \vee x = 2 \end{aligned}$$

[1]

$x$	$-\infty$		0		2		$+\infty$
$f'(x)$		+		+	0	+	
$f''(x)$		+	0	-	0	+	
$f$	0	$\nearrow$	$\frac{5}{e}$	$\nearrow$	$e$	$\nearrow$	$+\infty$
$G_f$		$\cup$	P.I.	$\cap$	P.I. (*)	$\cup$	

(\*) P.I. à tangente horizontale

La fonction  $f$  n'admet ni maximum, ni minimum.

Points d'inflexion :  $I_1(0; \frac{5}{e})$  et  $I_2(2; e)$ .

[2,5]

(d)  $f(0) = \frac{5}{e}$                        $f'(0) = \frac{2}{e}$

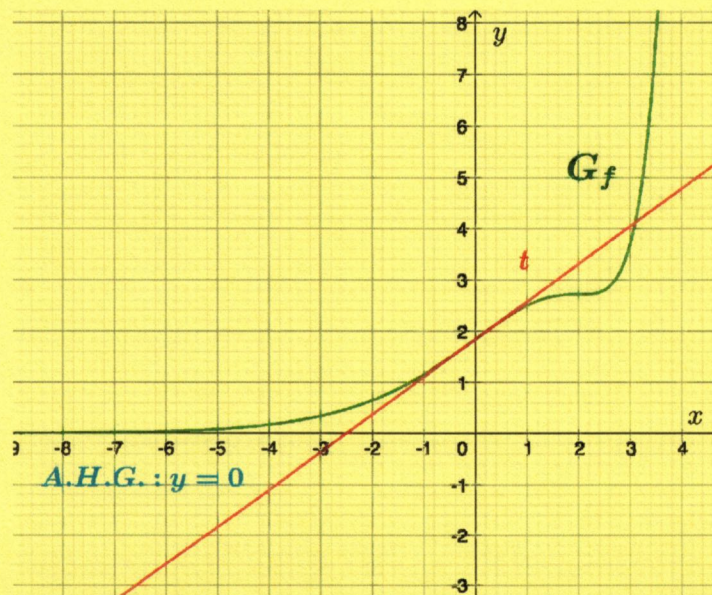
$$t \equiv y = f(0) + f'(0)(x - 0) \iff y = \frac{2}{e}x + \frac{5}{e}$$

[1]

(e)

$x$	-5	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	0,1	0,2	0,3	0,6	1,2	1,8	2,5	2,7	3,7	20,1

3/8



[3]

2)  $(\forall x \in \mathbb{R}) \quad f(x) > 0$  (d'après le graphique ou le tableau de variation).

$$\begin{aligned} \mathcal{A} &= \int_{-3}^3 f(x) dx \\ &= \left[ \frac{1}{2} (x^2 - 8x + 18) e^{x-1} \right]_{-3}^3 \\ &= \frac{1}{2} (9 - 24 + 18) e^2 - \frac{1}{2} (9 + 24 + 18) e^{-4} \\ &= \frac{3e^2}{2} - \frac{51}{2e^4} \\ &\approx 10,62 \text{ u.a.} \end{aligned}$$

[2]

Calcul à part :

$$\int (x^2 - 6x + 10) \cdot e^{x-1} dx$$

$$\begin{array}{ll} \text{ipp.} & u(x) = x^2 - 6x + 10 & v'(x) = e^{x-1} \\ & u'(x) = 2x - 6 & v(x) = e^{x-1} \end{array}$$

$$= (x^2 - 6x + 10) e^{x-1} - \int (2x - 6) e^{x-1} dx$$

$$\begin{array}{ll} \text{ipp.} & u(x) = 2x - 6 & v'(x) = e^{x-1} \\ & u'(x) = 2 & v(x) = e^{x-1} \end{array}$$

$$= (x^2 - 6x + 10) e^{x-1} - (2x - 6) e^{x-1} + \int 2 \cdot e^{x-1} dx$$

$$= (x^2 - 8x + 16) e^{x-1} + 2e^{x-1} + c$$

$$= (x^2 - 8x + 18) e^{x-1} + c \quad (c \in \mathbb{R})$$

[3]

## Question 4

[5+5+5=15 points]

$$1) I = \int_1^{e^2} \frac{3 \ln x - 2x^2 \ln^2 x}{x^3} dx = \int_1^{e^2} \left( \frac{3 \ln x}{x^3} - \frac{2 \ln^2 x}{x} \right) dx$$

Calculs à part :

$$\begin{aligned} \int \frac{\ln x}{x^3} dx & \quad \text{i.p.p.} \quad \begin{array}{l} u(x) = \ln x \\ u'(x) = \frac{1}{x} \end{array} \quad \begin{array}{l} v'(x) = \frac{1}{x^3} \\ v(x) = -\frac{1}{2x^2} \end{array} \\ &= -\frac{\ln x}{2x^2} + \int \frac{1}{2x^3} dx \\ &= -\frac{\ln x}{2x^2} + \frac{1}{2} \cdot \frac{1}{-2x^2} + c \\ &= -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + c \quad (c \in \mathbb{R}) \end{aligned}$$

$$\int \frac{\ln^2 x}{x} dx = \frac{\ln^3 x}{3} + c'$$

$$\begin{aligned} I &= \left[ -\frac{3 \ln x}{2x^2} - \frac{3}{4x^2} - \frac{2 \ln^3 x}{3} \right]_1^{e^2} \\ &= \frac{-3 \ln e^2}{2e^4} - \frac{3}{4e^4} - \frac{2 \ln^3(e^2)}{3} + \frac{3}{2} \ln 1 + \frac{3}{4} + \frac{2}{3} \ln^3 1 \\ &= \frac{-6}{2e^4} - \frac{3}{4e^4} - \frac{2[\ln(e^2)]^3}{3} + \frac{3}{4} \\ &= \frac{-3}{e^4} - \frac{3}{4e^4} - \frac{2 \cdot 8}{3} + \frac{3}{4} \\ &= -\frac{15}{4e^4} - \frac{55}{12} \quad (\approx -4,65) \end{aligned}$$

[5]

$$2) \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (1 + \cos^2 x) \cdot \sin^3 x dx$$

$$\begin{aligned} &= \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (1 + \cos^2 x) \cdot \sin^2 x \cdot \sin x dx \\ &= \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (1 + \cos^2 x) \cdot (1 - \cos^2 x) \cdot \sin x dx \\ &= \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (1 - \cos^4 x) \cdot \sin x dx \\ &= \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\sin x - \cos^4 x \cdot \sin x) dx \\ &= \left[ -\cos x + \frac{1}{5} \cos^5 x \right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \\ &= -\cos\left(\frac{5\pi}{6}\right) + \frac{1}{5} \cdot \left(\cos\frac{5\pi}{6}\right)^5 + \cos\frac{\pi}{6} - \frac{1}{5} \left(\cos\frac{\pi}{6}\right)^5 \\ &= \frac{\sqrt{3}}{2} + \frac{1}{5} \left(-\frac{\sqrt{3}}{2}\right)^5 + \frac{\sqrt{3}}{2} - \frac{1}{5} \left(\frac{\sqrt{3}}{2}\right)^5 \\ &= \sqrt{3} - \frac{2}{5} \left(\frac{9\sqrt{3}}{32}\right) \\ &= \frac{71\sqrt{3}}{80} \end{aligned}$$

[5]

$$\begin{aligned}
 3) \int_0^{\frac{1}{2}} \frac{8x^2 + 12x}{(4x^2 + 1)(4x + 1)} dx \\
 \frac{8x^2 + 12x}{(4x^2 + 1)(4x + 1)} &= \frac{ax + b}{4x^2 + 1} + \frac{c}{4x + 1} \\
 &= \frac{(ax + b)(4x + 1) + c(4x^2 + 1)}{(4x^2 + 1)(4x + 1)} \\
 &= \frac{(4a + 4c)x^2 + (a + 4b)x + (b + c)}{(4x^2 + 1)(4x + 1)}
 \end{aligned}$$

Par identification des coefficients :

$$\begin{cases} 4a + 4c = 8 \\ a + 4b = 12 \\ b + c = 0 \end{cases} \iff \begin{cases} a + c = 2 \\ a + 4b = 12 \\ b + c = 0 \end{cases} \iff \begin{cases} a + c = 2 \\ 5c = -10 \\ b = -c \end{cases} \iff \begin{cases} a = 4 \\ c = -2 \\ b = 2 \end{cases}$$

$$\text{D'où : } \frac{8x^2 + 12x}{(4x^2 + 1)(4x + 1)} = \frac{4x + 2}{4x^2 + 1} + \frac{-2}{4x + 1}$$

$$\begin{aligned}
 &\int_0^{\frac{1}{2}} \frac{8x^2 + 12x}{(4x^2 + 1)(4x + 1)} dx \\
 &= \int_0^{\frac{1}{2}} \left( \frac{4x}{4x^2 + 1} + \frac{2}{4x^2 + 1} + \frac{-2}{4x + 1} \right) dx \\
 &= \int_0^{\frac{1}{2}} \left( \frac{1}{2} \cdot \frac{8x}{4x^2 + 1} + \frac{2}{(2x)^2 + 1} - \frac{1}{2} \cdot \frac{4}{4x + 1} \right) dx \\
 &= \left[ \frac{1}{2} \cdot \ln |4x^2 + 1| + \text{Arctan}(2x) - \frac{1}{2} \ln |4x + 1| \right]_0^{\frac{1}{2}} \\
 &= \frac{1}{2} \ln 2 + \text{Arctan } 1 - \frac{1}{2} \ln 3 - \frac{1}{2} \ln 1 + \text{Arctan } 0 + \frac{1}{2} \ln 1 \\
 &= \frac{1}{2} \ln 2 - \frac{1}{2} \ln 3 + \frac{\pi}{4}
 \end{aligned}$$

[5]

**Question 5 (au choix)**

**[6+3=9 points]**

$$f(x) = 2x - 1 - 3 \ln \frac{x}{x + 2}$$

$$1) \text{ Conditions d'existence : } x + 2 \neq 0 \wedge \frac{x}{x + 2} > 0 \iff x < -2 \vee x > 0$$

$$\text{Dom } f = ] - \infty; -2[ \cup ] 0; +\infty[$$

- $\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} (2x - 1 - 3 \underbrace{\ln \frac{x}{x + 2}}_{\rightarrow 1}) = -\infty$
- $\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} (2x - 1 - 3 \underbrace{\ln \frac{x}{x + 2}}_{\rightarrow 1}) = +\infty$

Il n'y a ni A.H.G, ni A.H.D.

[2]

Posons  $y = 2x - 1$

$$\lim_{x \rightarrow \pm\infty} [f(x) - (2x - 1)] = \lim_{x \rightarrow \pm\infty} \left( -3 \ln \frac{x}{x+2} \right) = 0$$

Par conséquent,  $G_f$  admet comme A.O. la droite  $\Delta \equiv y = 2x - 1$ .

[2]

$$\blacksquare \lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^-} \left( 2x - 1 - 3 \underbrace{\ln \frac{x}{x+2}}_{\substack{\rightarrow +\infty \\ \rightarrow +\infty}} \right) = -\infty \quad \text{A.V. : } x = -2$$

[1]

$$\blacksquare \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \left( 2x - 1 - 3 \underbrace{\ln \frac{x}{x+2}}_{\substack{\rightarrow 0^+ \\ \rightarrow -\infty}} \right) = +\infty \quad \text{A.V. : } x = 0$$

[1]

2) Soit  $M(x; f(x)) \in G_f$  et  $P(x; y) \in \Delta$

$$f(x) - y = -3 \ln \frac{x}{x+2}$$

$$f(x) - y > 0 \iff -3 \ln \frac{x}{x+2} > 0$$

$$\iff \ln \frac{x}{x+2} < 0$$

$$\iff \frac{x}{x+2} < 1 \quad \text{car la fct ln est strict. croissante sur } \mathbb{R}_+^*$$

$$\iff \frac{x}{x+2} - 1 < 0$$

$$\iff \frac{-2}{x+2} < 0$$

$$\iff x + 2 > 0$$

$$\iff x > -2$$

$x$	$-\infty$	$-2$	$0$	$+\infty$
$f(x) - y$	$-$	$/$	$/$	$+$
	$\Delta/G_f$	$/$	$/$	$G_f/\Delta$

[3]

**Question 6 (au choix)**

[2+6+1=9 points]

$$f(x) = 2x - 1 + \frac{e^{2x}}{e^x + 1}$$

$$1) \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \left( \underbrace{2x - 1}_{\rightarrow -\infty} + \underbrace{\frac{e^{2x}}{e^x + 1}}_{\substack{\rightarrow 0 \\ \rightarrow 1}} \right) = -\infty, \text{ donc } G_f \text{ n'admet pas d'A.H.G.}$$

D'après ce qui précède :

$$\lim_{x \rightarrow -\infty} (f(x) - (2x - 1)) = \lim_{x \rightarrow -\infty} \frac{e^{2x}}{e^x + 1} = 0, \text{ donc } G_f \text{ admet l'A.O.G } \Delta \equiv y = 2x - 1.$$

[2]

$$2) \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \left( \underbrace{2x - 1}_{\rightarrow +\infty} + \underbrace{\frac{e^{2x}}{e^x + 1}}_{\rightarrow +\infty} \right).$$

Calcul à part :

$$\begin{aligned} & \lim_{x \rightarrow +\infty} \frac{e^{2x}}{e^x + 1} \quad \text{f.i. } \frac{\infty}{\infty} \\ &= \lim_{x \rightarrow +\infty} \frac{e^{2x}}{e^x(1 + e^{-x})} \\ &= \lim_{x \rightarrow +\infty} \frac{e^x}{1 + e^{-x}} \\ &= +\infty \end{aligned}$$

Donc  $\lim_{x \rightarrow +\infty} f(x) = +\infty$  et  $G_f$  n'admet pas d'A.H.D. [2]

$$\lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} \left( 2 - \frac{1}{x} + \frac{e^{2x}}{e^x + 1} \right).$$

Calcul à part :

$$\begin{aligned} & \lim_{x \rightarrow +\infty} \frac{\underbrace{e^{2x}}_{\rightarrow +\infty}}{\underbrace{x}_{\rightarrow +\infty}} \quad \text{f.i. } \frac{\infty}{\infty} \\ & \stackrel{H}{=} \lim_{x \rightarrow +\infty} \frac{2e^{2x} \cdot (e^x + 1) - e^{2x} \cdot e^x}{(e^x + 1)^2} \\ &= \lim_{x \rightarrow +\infty} \frac{e^{2x} \cdot (2e^x + 2 - e^x)}{(e^x + 1)^2} \\ &= \lim_{x \rightarrow +\infty} \frac{e^{2x} \cdot (e^x + 2)}{e^{2x} \cdot (1 + e^{-x})^2} \\ &= \lim_{x \rightarrow +\infty} \frac{e^x + 2}{(1 + e^{-x})^2} \\ &= +\infty \end{aligned}$$

Par conséquent :  $\lim_{x \rightarrow +\infty} \frac{f(x)}{x} = +\infty$  et  $G_f$  n'admet pas d'A.O.D. [4]

3) Position de  $G_f$  par rapport à  $\Delta \equiv y = 2x - 1$  :

Soit  $M(x; f(x)) \in G_f$  et  $P(x; y) \in \Delta$ .

$$f(x) - y = \frac{e^{2x}}{e^x + 1} > 0.$$

On en déduit :  $(\forall x \in \mathbb{R}) \quad G_f$  se situe au-dessus de  $\Delta$ . [1]