CALCUL INTEGRAL

Exercice 1

Calculez les primitives suivantes :

A) Calcul direct à partir des formules fondamentales.

1) $\int (5x-3)dx$ (sur $\mathbb{R}$)

2) $\int (2x^5 - \frac{3}{5}x^2 + 17x - 2,4)dx$ (sur $\mathbb{R}$)

3) $\int (7x^9 - x^6 + 4x^5 - \frac{8x^3}{11} + 6x - 3)dx$ (sur $\mathbb{R}$)

4) $\int (\sqrt{t} - \frac{1}{t})\, dt$ (sur $\mathbb{R}_+^*$)

5) $\int (\frac{5}{x^4} - \frac{13}{2x^3} + \frac{8}{\sqrt{x}})dx$ (sur $\mathbb{R}_+^*$)

6) $\int (\frac{2x}{5x^7} - \sqrt[3]{x^5} + \frac{\sqrt{x}}{x^2})dx$ (sur $\mathbb{R}_+^*$)

7) $\int (\frac{7x^6 - 9x^4 + 11x^3 - x + 2}{x^5})dx$ (sur $\mathbb{R}_+^*$)

8) $\int (\frac{x^8 + 15x^7 - 29x^5 + ex^2 - \sqrt{x}}{5x^3})dx$ (sur $\mathbb{R}_+^*$)

9) $\int (z+5)\sqrt{z}\, dz$ (sur $\mathbb{R}_+^*$)

10) $\int (\frac{(1+3x)^2}{\sqrt{x}})dx$ (sur $\mathbb{R}_+^*$)

11) $\int x^2(x^3-5)^8 dx$ (sur $\mathbb{R}$)

12) $\int \frac{5x^2}{(2x^3-1)^4}dx$ (sur $[1, +\infty)$)

13) $\int (3x^2-7)^2\, dx$ (sur $\mathbb{R}$)

14) $\int \frac{7x}{\left(3x^2+5\right)^2}\,dx$ (sur $\mathbb{R}$)

15) $\int \frac{1}{2}y^3\sqrt{y^4+2}\,dy$ (sur $\mathbb{R}$)

16) $\int \frac{5}{1-s}\,ds$ (sur $[1,+\infty)$)

17) $\int \frac{5-5x}{4x^2-8x+7}\,dx$ (sur $\mathbb{R}$)

18) $\int (2x-1)(5-7x)\,dx$ (sur $\mathbb{R}$)

19) $\int \frac{10-4x}{x^2-5x+8}\,dx$ (sur $\mathbb{R}$)

20) $\int \frac{5}{x^2-4x+4}\,dx$ (sur $]2,+\infty)$)

21) $\int \frac{\sqrt[3]{x}}{\sqrt{x}}\,dx$ (sur $\mathbb{R}_0^+$)

22) $\int \frac{6x+3}{x^2+x+3}\,dx$ (sur $\mathbb{R}$)

23) $\int (2-16x)e^{4x^2-x+1}\,dx$ (sur $\mathbb{R}$)

24) $\int \frac{\sqrt{\ln x}}{x}\,dx$ (sur $[1,+\infty)$)

25) $\int e^{3y+1}\,dy$ (sur $\mathbb{R}$)

26) $\int \frac{3}{e^{5-2x}}\,dx$ (sur $\mathbb{R}$)

27) $\int \frac{e^{2x}-3e^x+7}{e^x}\,dx$ (sur $\mathbb{R}$)

28) $\int \sin(3x-1)\,dx$ (sur $\mathbb{R}$)

29) $\int (\cos 2x+5\sin x)\,dx$ (sur $\mathbb{R}$)

30) $\int 3^{2x+1}\,dx$ (sur $\mathbb{R}$)

31) $\int \left(\frac{2}{3}\right)^{1-x}\,dx$ (sur $\mathbb{R}$)

32) $\int 5xe^{1-7x^2}\,dx$ (sur $\mathbb{R}$)

33) $\int \left(\frac{1}{\cos^2 x} - \frac{2}{x^2+1}\right) dx$ (sur $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$)

34) $\int \left(7\sin(9-5x) - 2\cos\frac{x}{3}\right) dx$ (sur $\mathbb{R}$)

35) $\int \left(1+\tan^2 4x\right) dx$ (sur $\left]-\frac{\pi}{8}, \frac{\pi}{8}\right[$)

36) $\int \tan^2 x\,dx$ (sur $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$)

37) $\int \tan x\,dx$ (sur $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$)

38) $\int \frac{1}{x^2+4}\,dx$ (sur $\mathbb{R}$)

39) $\int \frac{5}{\sqrt{9-x^2}}\,dx$ (sur $]-3,3[$)

40) $\int \frac{2}{3\cos^2 5x}\,dx$ (sur $\left]-\frac{\pi}{10}, \frac{\pi}{10}\right[$)

41) $\int \left(\frac{1-2e^{3x-1}}{e^{5x+2}} - 2\right) dx$ (sur $\mathbb{R}$)

42) $\int 3x\sin 7x^2\,dx$ (sur $\mathbb{R}$)

43) $\int \frac{2x}{\cos^2\left(x^2-1\right)}\,dx$ (sur $]-1,1[$)

44) $\int \frac{A\sin x}{\sqrt{1-x^2}}\,dx$ (sur $]-1,1[$)

45) $\int \frac{dx}{\sqrt{4-9x^2}}$ (sur $\left]-\frac{2}{3}, \frac{2}{3}\right[$)

46) $\int \frac{3e^x}{5+2e^x}\,dx$ (sur $\mathbb{R}$)

47) $\int \frac{\cos x}{\sin^7 x}\,dx$ (sur $]0,\pi[$)

48) $\int \left(e^{x}-e^{-x}\right)^{2} dx$ (sur $\mathbb{R}$)

49) $\int \frac{1+\ln^{3} x}{2x} dx$ (sur $\mathbb{R}_{+}^{*}$)

50) $\int \frac{1+\sin x}{\cos^{2} x} dx$ (sur $\left]-\frac{\pi}{2},\frac{\pi}{2}\right[$)

51) $\int \frac{\ln 5x}{3x} dx$ (sur $\mathbb{R}_{+}^{*}$)

52) $\int \frac{6}{5-x} dx$ (sur $(-\infty,5[$)

53) $\int \left(\frac{5}{7\sqrt[3]{x}}+5^{2x-1}\right) dx$ (sur $\mathbb{R}_{+}^{*}$)

54) $\int \frac{e^{\tan x}}{\cos^{2} x} dx$ (sur $\left]-\frac{\pi}{2},\frac{\pi}{2}\right[$)

55) $\int \frac{1}{1+\cos x} dx$ (sur $\mathbb{R}$)

56) $\int \sin 2x \cdot e^{\cos^{2} x} dx$ (sur $\mathbb{R}$)

57) $\int \frac{1-\cos 2x}{1+\cos 2x} dx$ (sur $\mathbb{R}$)

B) Décomposition en éléments simples de fractions rationnelles

58) $\int \frac{4x-1}{x-2} dx$ (sur $]2,+\infty)$)

Déterminez d'abord les coefficients réels a et b tels que : $\frac{4x-1}{x-2}=a+\frac{b}{x-2}$

59) $\int \frac{3x^{2}-x+1}{x^{2}-x-6} dx$ (sur $]-2,3[$)

Déterminez d'abord les coefficients réels a, b et c tels que :

$$\frac{3x^{2}-x+1}{x^{2}-x-6}=a+\frac{b}{x+2}+\frac{c}{x-3}$$

60) $\displaystyle\int \frac{x^2+3x+4}{x^3-3x^2-x+3}\,dx$ (sur $]3,+\infty)$)

Déterminez d'abord les coefficients réels a, b et c tels que :

$$\frac{x^2+3x+4}{x^3-3x^2-x+3}=\frac{a}{x-3}+\frac{b}{x-1}+\frac{c}{x+1}$$

61) $\displaystyle\int \frac{x^2+2x-5}{x^2+2x+1}\,dx$ (sur $]-1,+\infty)$)

Déterminez d'abord les coefficients réels a et b tels que :

$$\frac{x^2+2x-5}{x^2+2x+1}=a+\frac{b}{(x+1)^2}$$

62) $\displaystyle\int \frac{x^2-2x+3}{x^3-x^2+x-1}\,dx$ (sur $]1,+\infty)$)

Déterminez d'abord les coefficients réels a, b et c tels que :

$$\frac{x^2-2x+3}{x^3-x^2+x-1}=\frac{a}{x-1}+\frac{b}{x^2+1}$$

63) $\displaystyle\int \frac{2x^2+5x-9}{x^3-3x^2+4}\,dx$ (sur $]2,+\infty)$)

Déterminez d'abord les coefficients réels a, b et c tels que :

$$\frac{2x^2+5x-9}{x^3-3x^2+4}=\frac{a}{x+1}+\frac{b}{(x-2)^2}+\frac{c}{x-2}$$

64) $\displaystyle\int \frac{2x^2+13x+25}{(x+4)^2}\,dx$ (sur $]-4,-\infty)$)

Déterminez d'abord les coefficients réels a, b et c tels que :

$$\frac{2x^2+13x+25}{x^2+8x+16}=a+\frac{b}{(x+4)^2}+\frac{c}{x+4}$$

65) $\displaystyle\int \frac{-x^2+2x-31}{x^3-3x^2+25x-75}\,dx$ (sur $]3,-\infty)$)

Déterminez d'abord les coefficients réels a et b tels que :

$$\frac{-x^2+2x-31}{x^3-3x^2+25x-75}=\frac{a}{x^2+25}+\frac{b}{x-3}$$

C) Produits de fonctions trigonométriques

66) $\int \cos^2 x \, dx$ (sur $\mathbb{R}$)

67) $\int \cos^3 x \, dx$ (sur $\mathbb{R}$)

68) $\int \cos^4 x \, dx$ (sur $\mathbb{R}$)

69) $\int \sin^4 x \cdot \cos^3 x \, dx$ (sur $\mathbb{R}$)

70) $\int \sin^7 x \cdot \cos x \, dx$ (sur $\mathbb{R}$)

71) $\int \sin x \cdot \cos 2x \, dx$ (sur $\mathbb{R}$)

72) $\int \cos x \cdot \cos 4x \, dx$ (sur $\mathbb{R}$)

73) $\int \sin 3x \cdot \sin 5x \, dx$ (sur $\mathbb{R}$)

74) $\int \sin 5x \cdot \cos^3 5x \, dx$ (sur $\mathbb{R}$)

D) Intégration par parties

75) $\int x \sin 3x \, dx$ (sur $\mathbb{R}$)

76) $\int \ln x \, dx$ (sur $\mathbb{R}_+^*$)

77) $\int x\sqrt{1+2x} \, dx$ (sur $\left]-\frac{1}{2}, +\infty\right[$)

78) $\int \left(7x^2 - 3x + 6\right) \ln x \, dx$ (sur $\mathbb{R}_+^*$)

79) $\int \frac{x}{\cos^2 x} \, dx$ (sur $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$)

80) $\int x^2 e^{5x} \, dx$ (sur $\mathbb{R}$)

81) $\int \frac{\ln x}{x^2} \, dx$ (sur $\mathbb{R}_+^*$)

82) $\int A \sin x \, dx$ (sur $]-1;1[$)

E) Mélanges

83) $\int \frac{\mathrm{A}\tan x}{1+x^2}\,dx$ (sur $\mathbb{R}$)

84) $\int \mathrm{A}\tan 2x\,dx$ (sur $\mathbb{R}$)

85) $\int \frac{2x-5}{\sqrt{4-x^2}}\,dx$ (sur $]-2,2[$)

86) $\int x(2x+1)^8\,dx$ (sur $\mathbb{R}$)

87) $\int \frac{\ln^9 x}{x}\,dx$ (sur $\mathbb{R}_+^*$)

88) $\int \frac{\sin 2x}{1+\sin^2 x}\,dx$ (sur $\mathbb{R}$)

89) $\int \frac{1}{x\ln x}\,dx$ (sur $]1,+\infty)$)

90) $\int x\ln x\,dx$ (sur $\mathbb{R}_+^*$)

91) $\int x\ln^2 x\,dx$ (sur $\mathbb{R}_+^*$)

92) $\int x^2\ln x\,dx$ (sur $\mathbb{R}_+^*$)

93) $\int \frac{x}{1+x^4}\,dx$ (sur $\mathbb{R}$)

94) $\int \frac{1+\tan^2 x}{\sqrt{1-\tan^2 x}}\,dx$ (sur $\left]-\frac{\pi}{4},\frac{\pi}{4}\right[$)

95) $\int \frac{\cos 2x}{1+\sin^2 2x}\,dx$ (sur $\mathbb{R}$)

96) $\int \frac{3x-7}{1+x^2}\,dx$ (sur $\mathbb{R}$)

97) $\int x\mathrm{A}\tan x\,dx$ (sur $\mathbb{R}$)

98) $\int \frac{\sin 2x-1}{\cos^2 x}\,dx$ (sur $\left]-\frac{\pi}{2},\frac{\pi}{2}\right[$)

99) $\int \frac{2x}{\sin^2 x}\,dx$ (sur $]0,\pi[$)

Exercice 2

Pour les fonctions suivantes, trouvez <u>la</u> primitive F telle que $F\left(x_0\right)=y_0$:

1) $f(x)=\dfrac{1}{1+\cos x}$, $\quad x_0=\dfrac{\pi}{3}$ et $y_0=5$

2) $f(x)=\dfrac{1-\cos 2x}{1+\cos 2x}$, $\quad x_0=\dfrac{\pi}{4}$ et $y_0=-7$

3) $f(x)=\cos x\cdot e^x$, $\quad x_0=0$ et $y_0=\dfrac{2}{3}$

Exercice 3

Calculez l'aire de la partie du plan délimitée par G_f, (Ox) et les droites d'équations $x=a$ et $x=b$ avec :

1) $f(x)=5x-6$, $a=-1$ et $b=3$.

2) $f(x)=\sqrt{2x}$, $a=0$ et $b=4$.

3) $f(x)=x^2-2x-3$, $a=-2$ et $b=5$.

4) $f(x)=\sin x$, $a=\dfrac{\pi}{2}$ et $b=\dfrac{3\pi}{2}$.

5) $f(x)=x^3-4x^2+x+6$, $a=-2$ et $b=1$.

6) $f(x)=\dfrac{1}{1-x}$, $a=2$ et $b=3$.

7) $f(x)=e^x$, $a=\ln 2$ et $b=\ln 3$.

8) $f(x)=\dfrac{1}{\sqrt{3-x^2}}$, $a=-1$ et $b=1$.

9) $f(x)=\dfrac{1}{3}x^3-\dfrac{5}{6}x^2-\dfrac{3}{2}x+3$, $a=-3$ et $b=4$.

10) $f(x)=\ln x$, $a=\dfrac{1}{e}$ et $b=e$.

11) $f(x)=\operatorname{A}\sin x$, $a=-\dfrac{1}{2}$ et $b=1$.

12) $f(x)=\dfrac{\ln x}{x}$, $a=\dfrac{1}{e}$ et $b=e$.

13) $f(x)=x^2\ln x$, $a=\dfrac{1}{\sqrt{e}}$ et $b=e$.

Exercice 4

Calculez l'aire de la surface délimitée par les courbes de f et g et les droites d'équations

$x=a$ et $x=b$ avec :

1) $f(x)=x^2+2$, $g(x)=x+1$, $a=-1$ et $b=1$.

2) $f(x)=\ln\left(x^2+1\right)$, $g(x)=\ln 2$, $a=-3$ et $b=2$

Montrez d'abord que $\dfrac{2x^2}{x^2+1}=a+\dfrac{b}{x^2+1}$ où a et b sont deux réels.

3) $f(x)=-\sqrt{x}$, $g(x)=-1$, $a=0$ et $b=4$.

4) $f(x)=\cos x$, $g(x)=\sin x$, $a=0$ et $b=\dfrac{\pi}{2}$.

5) $f(x)=x^2-2x+1$, $g(x)=\dfrac{x+1}{x-2}$, $a=\dfrac{5}{2}$ et $b=4$.

Montrez d'abord que $g(x)=a+\dfrac{b}{x-2}$ où a et b sont deux réels.

6) $f(x)=\operatorname{A}\tan x$, $g(x)=\operatorname{A}\sin x$, $a=-1$ et $b=1$ (pour l'intersection des deux courbes on pourra utiliser la V200).

7) $f(x)=\ln|x|$, $g(x)=e^{|x|}$, $a=-1$ et $b=-\dfrac{1}{e}$ (pour l'intersection des deux courbes on pourra utiliser la V200).

Exercice 5

Calculez l'aire de la surface fermée délimitée par les courbes de f et g avec :

1) $f(x)=2-x^2$ et $g(x)=\frac{1}{2}x+\frac{3}{2}$.

2) $f(x)=x^2+5$ et $g(x)=4x+5$.

3) $f(x)=x^2-4x$ et $g(x)=2x-\frac{1}{2}x^2$.

4) $f(x)=-\frac{1}{2}x^3-1$ et G_g est la droite passant par $A(-2,3)$ et l'origine.

5) $f(x)=x^3-3x+2$ et $g(x)=2x^2-2x$.

Exercice 6

Calculez le volume du solide engendré par la rotation autour de Ox de la surface délimitée par G_f, Ox et les droites d'équations $x=a$ et $x=b$ (figure !) avec :

1) $f(x)=2x+3$, $a=0$ et $b=2$.

2) $f(x)=1-x^2$, $a=-2$ et $b=1$.

3) $f(x)=\sqrt{x}+2$, $a=0$ et $b=2$.

4) $f(x)=e^x$, $a=-1$ et $b=1$.

5) $f(x)=\frac{2}{x}$, $a=-2$ et $b=-1$.

6) $f(x)=\frac{1}{\cos x}$, $a=\frac{\pi}{4}$ et $b=\frac{\pi}{3}$.

Exercice 7

Calculez le volume du solide engendré par la rotation autour de Ox de la surface fermée délimitée par les courbes de f et de g (figure) avec :

1) $f(x)=4-x^2$ et $g(x)=3$.

2) $f(x)=-\sqrt{4-x^2}$ et la droite d'équation $2x-3y-4=0$.

3) $f(x)=\left(\frac{1}{2}\right)^x$ et $g(x)=-\frac{3}{4}x+\frac{5}{4}$.

4) $f(x)=x^3+1$ et $g(x)=x+1$.

Exercice 8

Soit $f(x)=\dfrac{1}{x(1-\ln x)}$.

1) Etude de f : domaines, limites et branches infinies, dérivée et tableau de variation, concavité, courbe).

2) Trouvez l'équation de la tangente à la courbe issue de l'origine.

3) Soit $\lambda \in [1,e[$, calculez l'aire $A(\lambda)$ de la surface du plan délimitée par G_f et les droites d'équations $x=1$, $x=\lambda$ et $y=0$. Déterminez λ pour que cette aire soit égale à $\ln 2$.

Exercice 9

Soit $f(x)=4x\cdot e^{-\frac{1}{2}x}$.

1) Etude de f : domaines, limites et branches infinies, dérivée et tableau de variation, concavité, courbe).

2) Soit $\lambda \geq 0$, calculez l'aire $A(\lambda)$ de la surface du plan délimitée par G_f, Ox et les droites d'équations $x=0$ et $x=\lambda$ ainsi que le volume $V(\lambda)$ du solide engendré par la rotation autour de Ox de cette surface.

3) Calculez $\lim\limits_{\lambda\to+\infty} A(\lambda)$ et $\lim\limits_{\lambda\to+\infty} V(\lambda)$.

Exercice 1 (corrigé)

1) $\int (5x-3)\,dx = 5\frac{1}{2}x^2 - 3x + k = \frac{5}{2}x^2 - 3x + k$

2) $\int \left(2x^5 - \frac{3}{5}x^2 + 17x - 2{,}4\right)dx = \cancel{2}\cdot\frac{1}{\cancel{6}_3}x^6 - \frac{3}{5}\cdot\frac{1}{3}x^3 + 17\cdot\frac{1}{2}x^2 - 2{,}4x + k$

$= \frac{1}{3}x^6 - \frac{1}{5}x^3 + \frac{17}{2}x^2 - 2{,}4x + k$

3) $\int \left(7x^9 - x^6 + 4x^5 - \frac{8}{11}x^3 + 6x - 3\right)dx$

$= 7\cdot\frac{1}{10}x^{10} - \frac{1}{7}x^7 + 4\cdot\frac{1}{6}x^6 - \frac{8}{11}\cdot\frac{1}{4}x^4 + 6\cdot\frac{1}{2}x^2 - 3x + k$

$= \frac{7}{10}x^{10} - \frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{2}{11}x^4 + 3x^2 - 3x + k$

4) $\int \left(\sqrt{t} - \frac{1}{t}\right)dt = \int \left(t^{\frac{1}{2}} - \frac{1}{t}\right)dt = \frac{1}{\frac{3}{2}}t^{3/2} - \ln|t| + k$

$= \frac{2}{3}\sqrt{t^3} - \ln|t| + k$

5) $\int \left(\frac{5}{x^4} - \frac{13}{2x^3} + \frac{8}{\sqrt{x}}\right)dx = \int \left(5x^{-4} - \frac{13}{2}x^{-3} + 8x^{-\frac{1}{2}}\right)dx$

$= 5\cdot\frac{1}{-3}x^{-3} - \frac{13}{2}\cdot\frac{1}{-2}x^{-2} + 8\frac{1}{\frac{1}{2}}x^{\frac{1}{2}} + k$

$= -\frac{5}{3x^3} + \frac{13}{4x^2} + 16\sqrt{x} + k$

6) $\int \left(\frac{2x}{5x^7} - \sqrt[3]{x^5} + \frac{\sqrt{x}}{x^2}\right)dx = \int \left(\frac{2}{5}x^{1-7} - x^{\frac{5}{3}} + x^{\frac{1}{2}-2}\right)dx$

$= \int \left(\frac{2}{5}x^{-6} - x^{\frac{5}{3}} + x^{-\frac{3}{2}}\right)dx$

$= \frac{2}{5}\cdot\frac{1}{-5}x^{-5} - \frac{1}{\frac{8}{3}}x^{\frac{8}{3}} + \frac{1}{-\frac{1}{2}}x^{-\frac{1}{2}} + k$

$= -\frac{2}{25x^5} - \frac{3}{8}\sqrt[3]{x^8} - \frac{2}{\sqrt{x}} + k$

7) $\int \frac{7x^6 - 9x^4 + 11x^3 - x + 2}{x^5}dx = \int \left(\frac{7x^6}{x^5} - \frac{9x^4}{x^5} + \frac{11x^3}{x^5} - \frac{x}{x^5} + \frac{2}{x^5}\right)dx$

$= \int \left(7x - 9\cdot\frac{1}{x} + 11x^{-2} - x^{-4} + 2x^{-5}\right)dx$

$= 7\cdot\frac{1}{2}x^2 - 9\ln|x| + 11\frac{1}{-1}x^{-1} - \frac{1}{-3}x^{-3} + 2\cdot\frac{1}{-4}x^{-4} + k$

$= \frac{7}{2}x^2 - 9\ln|x| - \frac{11}{x} + \frac{1}{3x^3} - \frac{1}{2x^4} + k$

8) $\displaystyle\int \frac{x^8+15x^7-29x^5+2x^2-\sqrt{x}}{5x^3}\,dx$

$\displaystyle = \int\left(\frac{x^{\cancel{8}5}}{\cancel{5x^3}} + \frac{{}^{3}\cancel{15}x^{\cancel{7}4}}{\cancel{5}\cancel{x^3}} - \frac{29x^{\cancel{5}2}}{5\cancel{x^3}} + \frac{2\cancel{x^2}}{5x^{\cancel{3}}} - \frac{x^{\frac{1}{2}}}{5x^3}\right)dx$

$\displaystyle = \int\left(\frac{1}{5}x^5 + 3x^4 - \frac{29}{5}x^2 + \frac{2}{5}\cdot\frac{1}{x} - \frac{1}{5}x^{-\frac{5}{2}}\right)dx \qquad \left(\frac{1}{2}-3=-\frac{5}{2}\right)$

$\displaystyle = \frac{1}{5}\cdot\frac{1}{6}x^6 + 3\cdot\frac{1}{5}x^5 - \frac{29}{5}\cdot\frac{1}{3}x^3 + \frac{2}{5}\cdot\ln|x| - \frac{1}{5}\cdot\frac{1}{-\frac{3}{2}}x^{-\frac{3}{2}} + k$

$\displaystyle = \frac{1}{30}x^6 + \frac{3}{5}x^5 - \frac{29}{15}x^3 + \frac{2}{5}\ln|x| + \frac{2}{15\sqrt{x^3}} + k$

9) $\displaystyle\int (z+5)\sqrt{z}\,dz = \int\left(z\cdot z^{\frac{1}{2}} + 5\cdot z^{\frac{1}{2}}\right)dz = \int\left(z^{\frac{3}{2}} + 5z^{\frac{1}{2}}\right)dz$

$\displaystyle = \frac{1}{\frac{5}{2}}z^{\frac{5}{2}} + 5\cdot\frac{1}{\frac{3}{2}}z^{\frac{3}{2}} + k$

$\displaystyle = \frac{2}{5}\sqrt{z^5} + \frac{10}{3}\sqrt{z^3} + k$

10) $\displaystyle\int \frac{(1+3x)^2}{\sqrt{x}}\,dx = \int \frac{1+6x+9x^2}{\sqrt{x}}\,dx = \int\left(\frac{1}{\sqrt{x}} + \frac{6x}{\sqrt{x}} + \frac{9x^2}{\sqrt{x}}\right)dx$

$\displaystyle = \int\left(x^{-\frac{1}{2}} + 6\cdot x^{\frac{1}{2}} + 9x^{\frac{3}{2}}\right)dx$

$\displaystyle = \frac{1}{\frac{1}{2}}x^{\frac{1}{2}} + 6\cdot\frac{1}{\frac{3}{2}}x^{\frac{3}{2}} + 9\cdot\frac{1}{\frac{5}{2}}x^{\frac{5}{2}} + k$

$\displaystyle = 2\sqrt{x} + 4\sqrt{x^3} + \frac{18}{5}\sqrt{x^5} + k$

11) $\displaystyle\int x^2(x^3-5)^8\,dx$

$\displaystyle = \frac{(x^3-5)^9}{27} + k$

posons : $u = x^3-5$

alors : $u' = 3x^2 \Leftrightarrow \frac{1}{3}u' = x^2$

$f = \frac{1}{3}u'\cdot u^8$

$F = \frac{1}{3}\cdot\frac{1}{9}u^9 + k = \frac{u^9}{27} + k$

12) $\displaystyle\int \frac{5x^2}{(2x^3-1)^4}\,dx$

$\displaystyle = -\frac{5}{18(2x^3-1)^3} + k$

posons : $u = 2x^3-1$

alors : $u' = 6x^2 \Leftrightarrow \frac{5}{6}u' = 5x^2$

$f = \frac{5}{6}u'u^{-4}$

$F = \frac{5}{6}\cdot\frac{1}{-3}u^{-3} + k$

$\displaystyle = -\frac{5}{18u^3} + k$

13) $\int (3x^2-7)^2\,dx$

$= \int (9x^4-42x^2+49)\,dx$

$= 9\cdot\frac{1}{5}x^5 - \overset{14}{\cancel{42}}\cdot\frac{1}{\cancel{3}}x^3+49x+k$

$= \frac{9}{5}x^5-14x^3+49x+k$

14) $\int \frac{7x}{(3x^2+5)^2}\,dx$

posous: $u=3x^2+5$

alors: $u'=6x \Leftrightarrow \frac{7}{6}u'=7x$

$f=\frac{7}{6}u'\cdot u^{-2}$

$F=\frac{7}{6}\cdot\frac{1}{-1}u^{-1}+k = -\frac{7}{6u}+k$

$= -\frac{7}{6(3x^2+5)}+k$

15) $\int \frac{1}{2}y^3\sqrt{y^4+2}\,dy$

posous: $u=y^4+2$

alors: $u'=4y^3 \Leftrightarrow \frac{1}{8}u'=\frac{1}{2}y^3$

$f=\frac{1}{8}u'\cdot u^{\frac{1}{2}}$

$F=\frac{1}{8}\cdot\frac{1}{\frac{3}{2}}\cdot u^{\frac{3}{2}}+k$

$=\frac{1}{12}\sqrt{u^3}+k$

$=\frac{1}{12}\sqrt{(y^4+2)^3}+k$

16) $\int \frac{5}{1-s}\,ds$

posous: $u=1-s$

alors: $u'=-1 \Leftrightarrow -5u'=5$

$f=-5\frac{u'}{u}$

$F=-5\ln|u|+k$

$=-5\ln|1-s|+k$

17) $\int \frac{5-5x}{4x^2-8x+7}\,dx$

posous: $u=4x^2-8x+7$

alors: $u'=8x-8=8(x-1)$

$\Leftrightarrow -\frac{5}{8}u'=-5(x-1)=5-5x$

$f=-\frac{5}{8}\frac{u'}{u}$

$F=-\frac{5}{8}\ln|u|+k$

$=-\frac{5}{8}\ln|4x^2-8x+7|+k$

18) $\int (2x-1)(5-7x)\,dx$

$=\int (10x-14x^2-5+7x)\,dx$

$=\int (-14x^2+17x-5)\,dx$

$=-14\cdot\frac{1}{3}x^3+17\cdot\frac{1}{2}x^2-5x+k = -\frac{14}{3}x^3+\frac{17}{2}x^2-5x+k$

19) $\int \frac{10-4x}{x^2-5x+8}\,dx$

posons : $u = x^2-5x+8$

alors : $u' = 2x-5 \Leftrightarrow -2\cdot u' = -4x+10$

$f = -2\,\frac{u'}{u}$

$F = -2\ln|u| + k$

$= -2\ln|x^2-5x+8| + k$

20) $\int \frac{5}{x^2-4x+4}\,dx$

$= \int \frac{5}{(x-2)^2}\,dx$

posons : $u = x-2$

alors : $u' = 1 \Leftrightarrow 5u' = 5$

$f = 5u'u^{-2}$

$F = 5\cdot\frac{1}{-1}u^{-1} + k = -\frac{5}{u} + k$

$= -\frac{5}{x-2} + k$

21) $\int \frac{\sqrt[3]{x}}{\sqrt{x}}\,dx = \int \frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}}\,dx = \int x^{\frac{1}{3}-\frac{1}{2}}\,dx = \int x^{-\frac{1}{6}}\,dx = \frac{1}{\frac{5}{6}}x^{\frac{5}{6}} + k$

$= \frac{6}{5}\sqrt[6]{x^5} + k$

22) $\int \frac{6x+3}{x^2+x+3}\,dx$

posons : $u = x^2+x+3$

alors : $u' = 2x+1 \Leftrightarrow 3\cdot u' = 6x+3$

$f = 3\,\frac{u'}{u}$

$F = 3\ln|u| + k$

$= 3\ln|x^2+x+3| + k$

23) $\int (2-16x)e^{4x^2-x+1}\,dx$

posons : $u = 4x^2-x+1$

alors : $u' = 8x-1 \Leftrightarrow -2u' = -16x+2$

$f = -2u'e^u$

$F = -2e^u + k$

$= -2\,e^{4x^2-x+1} + k$

24) $\int \frac{\sqrt{\ln x}}{x}\,dx$

posons : $u = \ln x$

alors : $u' = \frac{1}{x}$

$f = u'\cdot\sqrt{u} = u'\cdot u^{\frac{1}{2}}$

$F = \frac{1}{\frac{3}{2}}u^{\frac{3}{2}} + k = \frac{2}{3}\sqrt{u^3} + k$

$= \frac{2}{3}\sqrt{\ln^3 x} + k$

25) $\int e^{3y+1}\,dy$

posons : $u = 3y+1$

alors : $u' = 3 \Leftrightarrow \frac{1}{3}u' = 1$

$f = \frac{1}{3}u'e^u$

$F = \frac{1}{3}e^u + k$

$= \frac{1}{3}e^{3y+1} + k$

26) $\int \frac{3}{e^{5-2x}}dx = \int 3e^{2x-5}dx$

$= \frac{3}{2}e^{2x-5} + k$

posons : $u = 2x-5$
alors : $u' = 2 \Leftrightarrow \frac{3}{2}u' = 3$
$f = \frac{3}{2}u'e^u$
$F = \frac{3}{2}e^u + k$

27) $\int \frac{e^{2x} - 3e^x + 7}{e^x}dx = \int \left(\frac{e^{2x}}{e^x} - 3\frac{e^x}{e^x} + \frac{7}{e^x}\right)dx$

$= \int \left(e^x - 3 + 7\cdot e^{-x}\right)dx$ $\rightarrow$ $\begin{cases} u = -x \\ u' = -1 \Leftrightarrow -7u' = 7 \\ f = -7u'e^u \\ F = -7e^u + k \end{cases}$

$= e^x - 3x - 7e^{-x} + k$

28) $\int \sin(3x-1)\,dx$

$= -\frac{1}{3}\cos(3x-1) + k$

posons : $u = 3x-1$
alors : $u' = 3 \Leftrightarrow \frac{1}{3}u' = 1$
$f = \frac{1}{3}u'\sin u$
$F = -\frac{1}{3}\cos u + k$

29) $\int \left(\cos 2x + 5\sin x\right)dx$ $\rightarrow$ $\begin{cases} u = 2x \\ u' = 2 \Leftrightarrow \frac{1}{2}u' = 1 \\ f = \frac{1}{2}u'\cos u \\ F = \frac{1}{2}\sin u + k \end{cases}$

$= \frac{1}{2}\sin 2x - 5\cos x + k$

30) $\int 3^{2x+1}dx = \int e^{(2x+1)\ln 3}dx$

$= \frac{1}{2\ln 3}e^{(2x+1)\ln 3} + k$

$= \frac{1}{2\ln 3}\cdot 3^{2x+1} + k$

posons $u = (2x+1)\ln 3$
alors $u' = 2\ln 3$
$\Leftrightarrow \frac{1}{2\ln 3}u' = 1$
$f = \frac{1}{2\ln 3}u'e^u$
$F = \frac{1}{2\ln 3}e^u + k$

31) $\int \left(\frac{2}{3}\right)^{1-x}dx = \int e^{(1-x)\ln\frac{2}{3}}dx = \frac{1}{\ln\frac{3}{2}}e^{(1-x)\ln\frac{2}{3}} + k = \frac{1}{\ln\frac{3}{2}}\left(\frac{2}{3}\right)^{1-x} + k$

posons : $u = (1-x)\ln\frac{2}{3}$
alors : $u' = -\ln\frac{2}{3} = \ln\frac{3}{2} \Leftrightarrow \frac{1}{\ln\frac{3}{2}}u' = 1$
$f = \frac{1}{\ln\frac{3}{2}}u'e^u$
$F = \frac{1}{\ln\frac{3}{2}}e^u + k$

32) $\int 5x\, e^{1-7x^2} dx = -\frac{5}{14} e^{1-7x^2} + k$

posons : $u = 1-7x^2$

alors : $u' = -14x \Leftrightarrow -\frac{5}{14}u' = 5x$

$f = -\frac{5}{14} u' e^u$

$F = -\frac{5}{14} e^u + k$

33) $\int \left(\frac{1}{\cos^2 x} - \frac{2}{x^2+1}\right) dx = \int \frac{1}{\cos^2 x} dx - 2 \cdot \int \frac{1}{x^2+1} dx = \tan x - 2 \operatorname{Arctan} x + k$

34) $\int \left(\underbrace{7 \sin(9-5x)} \underbrace{- 2\cos \frac{x}{3}}\right) dx = \frac{7}{5} \cos(9-5x) - 6 \sin \frac{x}{3} + k$

pos. $u = 9-5x$	$u = \frac{x}{3}$
alors $u' = -5 \Leftrightarrow -\frac{7}{5}u' = 7$	$u' = \frac{1}{3} \Leftrightarrow -6u' = -2$
$f = -\frac{7}{5} u' \sin u$	$f = -6u' \cos u$
$F = \frac{7}{5} \cos u$	$F = -6 \sin u$

35) $\int (1+\tan^2 4x)\, dx = \frac{1}{4} \tan 4x + k$

posons : $u = 4x$

alors : $u' = 4 \Leftrightarrow \frac{1}{4}u' = 1$

$f = \frac{1}{4} u' (1+\tan^2 u)$

$F = \frac{1}{4} \tan u + k$

36) $\int \tan^2 x\, dx = \int (1+\tan^2 x - 1)\, dx = \int (1+\tan^2 x) dx - \int 1\, dx$

$= \tan x - x + k$

37) $\int \tan x\, dx = \int \frac{\sin x}{\cos x} dx = -\ln|\cos x| + k$

posons : $u = \cos x$

alors : $u' = -\sin x \Leftrightarrow -u' = \sin x$

$f = -\frac{u'}{u}$

$F = -\ln|u| + k$

38) $\int \frac{1}{x^2+4} dx = \int \frac{1}{4\left(\frac{x^2}{4}+1\right)} dx = \frac{1}{4} \int \frac{1}{\left(\frac{x}{2}\right)^2+1} dx$

posons : $u = \frac{x}{2}$

alors : $u' = \frac{1}{2} \Leftrightarrow 2u' = 1$

$f = 2\,\frac{u'}{u^2+1}$

$F = 2\,\text{Atan}\,u + k$

$\int \frac{1}{x^2+4}\,dx = \frac{1}{4}\cdot 2\,\text{Atan}\,\frac{x}{2} + k = \frac{1}{2}\,\text{Atan}\,\frac{x}{2} + k$

39) $\int \frac{5}{\sqrt{9-x^2}}\,dx = \int \frac{5}{\sqrt{9(1-\frac{x^2}{9})}}\,dx = \int \frac{5}{3\sqrt{1-(\frac{x}{3})^2}}\,dx = 5\,\text{Asin}\,\frac{x}{3} + k$

posons : $u = \frac{x}{3}$

alors : $u' = \frac{1}{3} \Leftrightarrow 5u' = \frac{5}{3}$

$f = 5\,\frac{u'}{\sqrt{1-u^2}}$

$F = 5\,\text{Asin}\,u + k$

40) $\int \frac{2}{3\cos^2 5x}\,dx = \frac{2}{3}\int \frac{1}{\cos^2 5x}\,dx = \frac{2}{3}\cdot\frac{1}{5}\tan 5x + k$

$= \frac{2}{15}\tan 5x + k$

posons : $u = 5x$

alors : $u' = 5 \Leftrightarrow \frac{1}{5}u' = 1$

$f = \frac{1}{5}\,\frac{u'}{\cos^2 u}$

$F = \frac{1}{5}\tan u + k$

41) $\int \left(\frac{1-2e^{3x-1}}{e^{5x+2}} - 2\right)dx = \int \left(\frac{1}{e^{5x+2}} - 2\,\frac{e^{3x-1}}{e^{5x+2}} - 2\right)dx$

$= \int \left(\underbrace{e^{-5x-2}} \underbrace{-2e^{-2x-3}} - 2\right)dx = -\frac{1}{5}e^{-5x-2} + e^{-2x-3} - 2x + k$

$u = -5x-2$	$u = -2x-3$
$u' = -5 \Leftrightarrow -\frac{1}{5}u' = 1$	$u' = -2$
$f = -\frac{1}{5}u'e^u$	$f = u'e^u$
$F = -\frac{1}{5}e^u$	$F = e^u$

42) $\int 3x \sin 7x^2 \, dx = -\frac{3}{14}\cos 7x^2 + k$

posons : $u = 7x^2$

alors : $u' = 14x \Leftrightarrow \frac{3}{14}u' = 3x$

$f = \frac{3}{14} u' \sin u$

$F = -\frac{3}{14}\cos u + k$

43) $\int \frac{2x}{\cos^2(x^2-1)} dx = \tan(x^2-1) + k$

posons : $u = x^2 - 1$

alors : $u' = 2x$

$f = \frac{u'}{\cos^2 u}$

$F = \tan u + k$

44) $\int \frac{\text{Asin}\, x}{\sqrt{1-x^2}} dx = \frac{1}{2}\,\text{Asin}^2 x + k$

posons : $u = \text{Asin}\, x$

alors : $u' = \frac{1}{\sqrt{1-x^2}}$

$f = u'u$

$F = \frac{1}{2}u^2 + k$

45) $\int \frac{dx}{\sqrt{4-9x^2}} = \int \frac{1}{\sqrt{4(1-\frac{9x^2}{4})}} dx = \int \frac{1}{2\sqrt{1-\left(\frac{3x}{2}\right)^2}} dx = \frac{1}{3}\text{Asin}\frac{3x}{2} + k$

posons : $u = \frac{3x}{2}$

alors : $u' = \frac{3}{2} \Leftrightarrow \frac{1}{3}u' = \frac{1}{2}$

$f = \frac{1}{3}\frac{u'}{\sqrt{1-u^2}}$

$F = \frac{1}{3}\text{Asin}\, u + k$

46) $\int \frac{3e^x}{5+2e^x} dx = \frac{3}{2}\ln|5+2e^x| + k$

posons : $u = 5 + 2e^x$

alors : $u' = 2e^x \Leftrightarrow \frac{3}{2}u' = 3e^x$

$f = \frac{3}{2}\frac{u'}{u}$

$F = \frac{3}{2}\ln|u| + k$

47) $\int \frac{\cos x}{\sin^7 x}\,dx = -\frac{1}{6\sin^6 x} + k$

posons : $u = \sin x$
alors : $u' = \cos x$

$f = \frac{u'}{u^7} = u' \cdot u^{-7}$

$F = \frac{1}{-6} u^{-6} + k = -\frac{1}{6u^6} + k$

48) $\int (e^x - e^{-x})^2\,dx = \int (\underbrace{e^{2x}} - 2 + \underbrace{e^{-2x}})\,dx = \frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + k$

$u = 2x$	$u = -2x$
$u' = 2 \Leftrightarrow \frac{1}{2}u' = 1$	$u' = -2 \Leftrightarrow -\frac{1}{2}u' = 1$
$f = \frac{1}{2}u'e^u$	$f = -\frac{1}{2}u'e^u$
$F = \frac{1}{2}e^u$	$F = -\frac{1}{2}e^u$

49) $\int \frac{1 + \ln^3 x}{2x}\,dx = \int \frac{1}{2x}\,dx + \int \frac{\ln^3 x}{2x}\,dx$

$= \frac{1}{2}\int \frac{1}{x}\,dx + \frac{1}{2}\underbrace{\int \frac{\ln^3 x}{x}\,dx}$

posons $u = \ln x$
alors $u' = \frac{1}{x}$

$f = u' \cdot u^3$

$F = \frac{1}{4}u^4 + k$

$= \frac{1}{2}\ln|x| + \frac{1}{2}\cdot\frac{1}{4}\ln^4 x + k$

$= \frac{1}{2}\ln|x| + \frac{1}{8}\ln^4 x + k$

50) $\int \frac{1 + \sin x}{\cos^2 x}\,dx = \int \frac{1}{\cos^2 x}\,dx + \underbrace{\int \frac{\sin x}{\cos^2 x}\,dx} = \tan x + \frac{1}{\cos x} + k$

posons : $u = \cos x$
alors : $u' = -\sin x \Leftrightarrow -u' = \sin x$

$f = -\frac{u'}{u^2} = -u'u^{-2}$

$F = -\frac{1}{-1}u^{-1} + k$

$= \frac{1}{u} + k$

51) $\int \frac{\ln 5x}{3x}\,dx = \frac{1}{6}\ln^2 5x + k$

posons : $u = \ln 5x$
alors : $u' = \frac{5}{5x} = \frac{1}{x} \Leftrightarrow \frac{1}{3}u' = \frac{1}{3x}$

$f = \frac{1}{3}u' \cdot u$

$F = \frac{1}{3}\cdot\frac{1}{2}u^2 + k = \frac{1}{6}u^2 + k$

52) $\int \frac{6}{5-x}\,dx = -6\ln|5-x|+k$

posons : $u = 5-x$
alors : $u' = -1 \Leftrightarrow -6u' = 6$
$f = -6\frac{u'}{u}$
$F = -6\ln|u|+k$

53) $\int \left(\frac{5}{7\sqrt[3]{x}} + 5^{2x-1}\right)dx = \frac{5}{7}\int x^{-\frac{1}{3}}dx + \underbrace{\int e^{(2x-1)\ln 5}dx}$

posons : $u = (2x-1)\ln 5$
alors : $u' = 2\ln 5 \Leftrightarrow \frac{1}{2\ln 5}u' = 1$
$f = \frac{1}{2\ln 5}u'e^u$
$F = \frac{1}{2\ln 5}e^u + k$

$= \frac{5}{7}\cdot\frac{1}{\frac{2}{3}}x^{\frac{2}{3}} + \frac{1}{2\ln 5}e^{(2x-1)\ln 5} + k$

$= \frac{15}{14}\sqrt[3]{x^2} + \frac{1}{2\ln 5}5^{2x-1} + k$

54) $\int \frac{e^{\tan x}}{\cos^2 x}\,dx = e^{\tan x} + k$

posons : $u = \tan x$
alors : $u' = \frac{1}{\cos^2 x}$
$f = u'\cdot e^u$
$F = e^u + k$

55) $\int \frac{1}{1+\cos x}\,dx = \int \frac{1}{2\cos^2\frac{x}{2}}\,dx = \tan\frac{x}{2} + k$

posons : $u = \frac{x}{2}$
alors : $u' = \frac{1}{2}$
$f = \frac{u'}{\cos^2 u}$
$F = \tan u + k$

56) $\int \sin 2x\, e^{\cos^2 x}\,dx = -e^{\cos^2 x} + k$

posons : $u = \cos^2 x$
alors : $u' = 2\cos x\cdot(-\sin x) = -2\cos x\sin x = -\sin 2x$
$\Leftrightarrow -u' = \sin 2x$
$f = -u'e^u$
$F = -e^u + k$

57) $\displaystyle\int \frac{1-\cos 2x}{1+\cos 2x}\,dx = \int \frac{2\sin^2 x}{2\cos^2 x}\,dx = \int \tan^2 x\,dx = \tan x - x + k$
(voir 36))

58) • $\dfrac{4x-1}{x-2} = a + \dfrac{b}{x-2}$ (*)

1re méthode : par identification

$(*) \Leftrightarrow \dfrac{4x-1}{x-2} = \dfrac{ax-2a+b}{x-2} \Leftrightarrow \begin{cases} a = 4 & (1) \\ -2a+b = -1 & (2) \end{cases}$

$(1) \to (2) : -8 + b = -1 \Leftrightarrow b = 7$

2e méthode : par calcul des limites

• $x \to +\infty$ $\quad \dfrac{4x-1}{x-2} \approx \dfrac{4\cancel{x}}{\cancel{x}} \to 4$

$a + \dfrac{b}{x-2} \to a + \left(\dfrac{b}{+\infty}\right) = a + 0 = a$ $\Big\}$ $\underline{a = 4}$

• $(*) \,|\cdot (x-2) \Leftrightarrow \underbrace{4x-1}_{\downarrow \, 7} = \underbrace{a(x-2)}_{\downarrow \, 0} + b$ $\qquad x \to 2$

donc $\underline{7 = b}$

• $\displaystyle\int \left(4 + \frac{7}{x-2}\right) dx = 4x + 7 \cdot \int \frac{1}{x-2}\,dx = 4x + 7\ln|x-2| + k$

$u = x-2$
$u' = 1$
$f = \dfrac{u'}{u}$
$F = \ln|u|$

59) • $\dfrac{3x^2 - x + 1}{x^2 - x - 6} = a + \dfrac{b}{x+2} + \dfrac{c}{x-3}$ (*)

vérification : $(x+2)(x-3) = x^2 - 3x + 2x - 6 \overset{?}{=} x^2 - x - 6$

1re méthode :

$a + \dfrac{b}{x+2} + \dfrac{c}{x-3} = \dfrac{a\cdot(x+2)(x-3) + b(x-3) + c(x+2)}{(x+2)(x-3)}$

$= \dfrac{ax^2 - ax - 6a + bx - 3b + cx + 2c}{x^2 - x - 6}$

$(*) \Leftrightarrow \begin{cases} a = 3 & (1) \\ -a + b + c = -1 & (2) \\ -6a - 3b + 2c = 1 & (3) \end{cases}$

$(1) \to (2) : -3 + b + c = -1 \Leftrightarrow b = 2 - c$ (4)

(1) et $(4) \to (3) : -18 - 6 + 3c + 2c = 1 \Leftrightarrow 5c = 25 \Leftrightarrow \underline{c = 5}$

donc $b = 2 - 5 = -3$

2^e^ méthode

• $x \to +\infty$: $\dfrac{3x^2-x+1}{x^2-x-6} = a + \dfrac{b}{x+2} + \dfrac{c}{x-3}$ d'où $\underline{3=a}$

$\approx \dfrac{3\cancel{x^2}}{\cancel{x^2}} \to 3 \qquad \left(\dfrac{b}{+\infty}\right)=0 \quad \left(\dfrac{c}{+\infty}\right)=0$

• $x \to -2$: $\dfrac{(3x^2-x+1)\cancel{(x+2)}}{\cancel{(x+2)}(x-3)} = a(x+2) + b + \dfrac{c(x+2)}{x-3}$ d'où $\underline{-3=b}$

$\dfrac{15}{-5}=-3 \qquad 0 \qquad 0$

• $x \to 3$: $\dfrac{(3x^2-x+1)\cancel{(x-3)}}{(x+2)\cancel{(x-3)}} = a(x-3) + \dfrac{b(x-3)}{x+2} + c$ d'où $\underline{5=c}$

$\hookrightarrow \dfrac{25}{5}=5 \qquad 0 \qquad 0$

• $\displaystyle\int \left(3 - \frac{3}{x+2} + \frac{5}{x-3}\right)dx = 3x - 3\cdot\ln|x+2| + 5\cdot\ln|x-3| + k$

6o) • $\dfrac{x^2+3x+4}{x^3-3x^2-x+3} = \dfrac{a}{x-3} + \dfrac{b}{x-1} + \dfrac{c}{x+1}$ (*)

vérification : $(x-3)(x-1)(x+1) = (x-3)(x^2-1) \overset{?}{=} x^3-x-3x^2+3$

1^re^ méthode

$\dfrac{a}{x-3} + \dfrac{b}{x-1} + \dfrac{c}{x+1} = \dfrac{a(x-1)(x+1)+b(x-3)(x+1)+c(x-3)(x-1)}{(x-3)(x-1)(x+1)}$

$= \dfrac{a(x^2-1)+b(x^2+\overset{-2x}{\cancel{x-3x}}-3)+c(x^2\overset{-4x}{\cancel{-x-3x}}+3)}{D}$

$= \dfrac{ax^2-a+bx^2-2bx-3b+cx^2-4cx+3c}{D}$

$(*) \Leftrightarrow \begin{cases} a+b+c=1 & (1) \\ -2b-4c=3 & (2) \\ -a-3b+3c=4 & (3) \end{cases}$

$(1)+(3)$: $-2b+4c=5$ (4)

$(4)+(2)$: $-4b=8 \Leftrightarrow \underline{b=-2}$

$\to (4)$: $4+4c=5 \Leftrightarrow \underline{c=\frac{1}{4}}$

$\to (1)$: $a-2+\frac{1}{4}=1 \Leftrightarrow a=3-\frac{1}{4} \Leftrightarrow \underline{a=\frac{11}{4}}$

2^e^ méthode

• $x \to 3$: $\dfrac{(x^2+3x+4)\cancel{(x-3)}}{\cancel{(x-3)}(x-1)(x+1)} = a + \dfrac{b(x-3)}{x-1} + \dfrac{c(x-3)}{x+1}$ d'où $\underline{a=\frac{11}{4}}$

$\to \frac{22}{8}=\frac{11}{4} \qquad \to 0 \qquad \to 0$

• $x \to 1$: $\dfrac{(x^2+3x+4)\cancel{(x-1)}}{(x-3)\cancel{(x-1)}(x+1)} = \dfrac{a(x-1)}{x-3} + b + \dfrac{c(x-1)}{x+1}$ d'où $\underline{b=-2}$

$\dfrac{8}{-4} = -2$; $\dfrac{a(x-1)}{x-3} \to 0$; $\dfrac{c(x-1)}{x+1} \to 0$

• $x \to -1$: $\dfrac{(x^2+3x+4)\cancel{(x+1)}}{(x-3)(x-1)\cancel{(x+1)}} = \dfrac{a(x+1)}{x-3} + \dfrac{b(x+1)}{x-1} + c$ d'où $\underline{c = \frac{1}{4}}$

$\dfrac{2}{8} = \dfrac{1}{4}$; $\dfrac{a(x+1)}{x-3} \to 0$; $\dfrac{b(x+1)}{x-1} \to 0$

• $\displaystyle\int \left(\frac{11}{4}\cdot\frac{1}{x-3} - 2\cdot\frac{1}{x-1} + \frac{1}{4}\cdot\frac{1}{x+1}\right)dx = \frac{11}{4}\ln|x-3| - 2\ln|x-1| + \frac{1}{4}\ln|x+1| + k$

6 1) • $\dfrac{x^2+2x-5}{x^2+2x+1} = a + \dfrac{b}{(x+1)^2}$ (*)

1^re^ méthode

$a + \dfrac{b}{(x+1)^2} = \dfrac{a(x+1)^2+b}{(x+1)^2} = \dfrac{a(x^2+2x+1)+b}{x^2+2x+1} = \dfrac{ax^2+2ax+a+b}{x^2+2x+1}$

$(*) \Leftrightarrow \begin{cases} a = 1 \\ 2a = 2 \\ a+b=-5 \end{cases} \Leftrightarrow \begin{cases} \underline{a=1} & (1) \\ a+b=-5 & (2) \end{cases}$

$(1) \to (2)$: $1+b=-5 \Leftrightarrow \underline{b=-6}$

2^e^ méthode

• $x \to +\infty$: $\dfrac{x^2+2x-5}{x^2+2x+1} = a + \dfrac{b}{(x+1)^2}$ d'où $\underline{a=1}$

$\approx \dfrac{x^2}{x^2} \to 1$; $\left(\dfrac{b}{+\infty}\right) = 0$

• $x \to -1$: $x^2+2x-5 = a(x+1)^2 + b$ d'où $\underline{b=-6}$

-6 ; $a(x+1)^2 \to 0$

• $\displaystyle\int \left(1 - 6\cdot(x+1)^{-2}\right)dx = x - 6\cdot\int (x+1)^{-2}dx = x + \frac{6}{x+1} + k$

$u = x+1$
$u' = 1$
$f = u'\cdot u^{-2}$
$F = \dfrac{1}{-1}u^{-1} = -\dfrac{1}{u}$

62) • $\dfrac{x^2-2x+3}{x^3-x^2+x-1} = \dfrac{a}{x-1} + \dfrac{b}{x^2+1}$ (*)

vérification : $(x-1)(x^2+1) \overset{?}{=} x^3+x-x^2-1$

1re méthode :

$\dfrac{a}{x-1} + \dfrac{b}{x^2+1} = \dfrac{a(x^2+1)+b(x-1)}{(x-1)(x^2+1)} = \dfrac{ax^2+a+bx-b}{D}$

(*) $\Leftrightarrow \begin{cases} a = 1 & (1) \\ b = -2 & (2) \\ a-b = 3 & (3) \end{cases}$

V(1) et (2) $\to$ (3) : $1-(-2) \overset{!}{=} 3$

2e méthode

• $x \to 1$: $\dfrac{(x^2-2x+3)\cancel{(x-1)}}{(x^2+1)\cancel{(x-1)}} = a + \dfrac{b(x-1)}{x^2+1}$ d'où $\underline{a=1}$

$\hookrightarrow \dfrac{2}{2} = 1$; $\dfrac{b(x-1)}{x^2+1} \to 0$

• $x \to i$: $\dfrac{(x^2-2x+3)\cancel{(x^2+1)}}{\cancel{(x^2+1)}\,(x-1)} = \dfrac{a(x^2+1)}{x-1} + b$ d'où $\underline{b=-2}$

$\hookrightarrow \dfrac{2-2i}{i-1} = \dfrac{-2\cancel{(i-1)}}{\cancel{i-1}} = -2$; $\dfrac{a(x^2+1)}{x-1} \to 0$

• $\displaystyle\int \left(\frac{1}{x-1} - 2\cdot\frac{1}{x^2+1}\right) dx = \ln|x-1| - 2\cdot \operatorname{Atan} x + k$

63) • $\dfrac{2x^2+5x-9}{x^3-3x^2+4} = \dfrac{a}{x+1} + \dfrac{b}{(x-2)^2} + \dfrac{c}{x-2}$ (*)

vérification : $(x+1)(x-2)^2 = (x+1)(x^2-4x+4)$
$= x^3-4x^2+4x+x^2-4x+4$
$\overset{?}{=} x^3-3x^2+4$

1re méthode

$\dfrac{a}{x+1} + \dfrac{b}{(x-2)^2} + \dfrac{c}{x-2} = \dfrac{a(x-2)^2+b(x+1)+c(x+1)(x-2)}{(x+1)(x-2)^2}$

$= \dfrac{a(x^2-4x+4)+bx+b+c(x^2-2x+x-2)}{D}$

$= \dfrac{ax^2-4ax+4a+bx+b+cx^2-cx-2c}{D}$

(*) $\Leftrightarrow \begin{cases} a+c = 2 & (1) \\ -4a+b-c = 5 & (2) \\ 4a+b-2c = -9 & (3) \end{cases}$

(3) − (2) : $8a-c = -14$ (4)

$(1)+(4)$: $9a = -12 \Leftrightarrow a = -\frac{12}{9} \Leftrightarrow \underline{a = -\frac{4}{3}}$

$\to (1)$: $-\frac{4}{3} + c = 2 \Leftrightarrow c = 2 + \frac{4}{3} \Leftrightarrow \underline{c = \frac{10}{3}}$

$\to (2)$: $\frac{16}{3} + b - \frac{10}{3} = 5 \Leftrightarrow 2 + b = 5 \Leftrightarrow \underline{b = 3}$

2e méthode

- $x \to -1$: $\frac{(2x^2+5x-9)\cancel{(x+1)}}{\cancel{(x+1)}(x-2)^2} = a + \frac{b(x+1)}{(x-2)^2} + \frac{c(x+1)}{x-2}$ d'où $\underline{a = -\frac{4}{3}}$

$\frac{b(x+1)}{(x-2)^2} \to 0$, $\frac{c(x+1)}{x-2} \to 0$

$\downarrow$

$\frac{-12}{9} = -\frac{4}{3}$

- $x \to 2$: $\frac{(2x^2+5x-9)\cancel{(x-2)^2}}{(x+1)\cancel{(x-2)^2}} = \frac{a(x-2)^2}{x+1} + b + c(x-2)$ donc $\underline{b = 3}$

$\frac{a(x-2)^2}{x+1} \to 0$, $c(x-2) \to 0$

$\downarrow$

$\frac{9}{3} = 3$

- $x \to +\infty$: $\frac{(2x^2+5x-9)\cancel{(x-2)}}{(x+1)(x-2)^{\cancel{2}}} = \frac{a(x-2)}{x+1} + \frac{b}{x-2} + c$

$\simeq \frac{2x^2}{x^2} \to 2$ $\qquad \simeq \frac{ax}{x} \to a = -\frac{4}{3}$ $\qquad \left(\frac{b}{\infty}\right) = 0$

d'où : $2 = -\frac{4}{3} + c \Leftrightarrow \underline{c = \frac{10}{3}}$

- $\int \left(-\frac{4}{3} \cdot \frac{1}{x+1} + 3 \cdot \underbrace{\frac{1}{(x-2)^2}} + \frac{10}{3} \cdot \frac{1}{x-2}\right) dx = -\frac{4}{3}\ln|x+1| - \frac{3}{x-2} + \frac{10}{3}\ln|x-2| + k$

$u = x-2$

$u' = 1$

$f = \frac{u'}{u^2} = u' \cdot u^{-2}$

$F = \frac{1}{-1} \cdot u^{-1} = -\frac{1}{u}$

64) • $\frac{2x^2+13x+25}{x^2+8x+16} = a + \frac{b}{(x+4)^2} + \frac{c}{x+4}$ $(*)$

vérification : $(x+4)^2 \overset{!}{=} x^2+8x+16$

1re méthode

$a + \frac{b}{(x+4)^2} + \frac{c}{x+4} = \frac{a(x+4)^2 + b + c(x+4)}{(x+4)^2} = \frac{a(x^2+8x+16) + b + cx + 4c}{D}$

$= \frac{ax^2 + 8ax + 16a + b + cx + 4c}{D}$

$(*) \Leftrightarrow \begin{cases} a = 2 & (1) \\ 8a + c = 13 & (2) \\ 16a + b + 4c = 25 & (3) \end{cases}$

$(1) \to (2) : 16 + c = 13 \Leftrightarrow \underline{c = -3}$

$\to (3) : 32 + b - 12 = 25 \rightsquigarrow \underline{b = 5}$

2e méthode

$\cdot\ x \to -4 : \underbrace{2x^2+13x+25}_{\downarrow\ 5} = \underbrace{a(x+4)^2}_{\downarrow\ 0} + b + \underbrace{c(x+4)}_{\downarrow\ 0}$ donc $\underline{b=5}$

$\cdot\ x \to \infty : \dfrac{2x^2+13x+25}{x^2+8x+16} = a + \dfrac{b}{(x+4)^2} + \dfrac{c}{x+4}$ donc $\underline{a=2}$

$\approx \dfrac{2x^2}{x^2} \to 2 \qquad \dfrac{b}{(x+4)^2} \to 0 \qquad \dfrac{c}{x+4} \to 0$

$\cdot\ (*) \,|\, (x+4) \Leftrightarrow \dfrac{2x^2+13x+25}{x+4} = 2(x+4) + \dfrac{5}{x+4} + c$

$\Leftrightarrow c = \dfrac{2x^2+13x+25-5-2(x^2+8x+16)}{x+4} = \dfrac{-3x-12}{x+4}$

$= \dfrac{-3\cancel{(x+4)}}{\cancel{x+4}} = \underline{-3 = c}$

$\bullet \displaystyle\int \left(2 + \frac{5}{(x+4)^2} - 3\cdot\frac{1}{x+4}\right)dx = 2x - \frac{5}{x+4} - 3\ln|x+4| + k$

$u = x+4$

$u' = 1$

$f = 5u'\cdot u^{-2}$

$F = \dfrac{5}{-1}\cdot u^{-1} = -\dfrac{5}{u}$

65) $\bullet\ \dfrac{-x^2+2x-31}{x^3-3x^2+25x-75} = \dfrac{a}{x^2+25} + \dfrac{b}{x-3} \quad (*)$

vérif.: $(x^2+25)(x-3) \overset{!}{=} x^3-3x^2+25x-75$

1re méthode

$\dfrac{a}{x^2+25} + \dfrac{b}{x-3} = \dfrac{a(x-3)+b(x^2+25)}{(x^2+25)(x-3)} = \dfrac{ax-3a+bx^2+25b}{D}$

$(*) \Leftrightarrow \begin{cases} \underline{b=-1} & (1) \\ \underline{a=2} & (2) \\ -3a+25b=-31 & (3) \end{cases}$

vérifions (3): $-3\cdot 2 + 25\cdot(-1) = -6-25 \overset{!}{=} -31$

2e méthode

$\cdot\ x \to 3 : \dfrac{-x^2+2x-31}{x^2+25} = \dfrac{a(x-3)}{x^2+25} + b$ donc $\underline{b=-1}$

$\hookrightarrow \dfrac{-34}{34} = -1 \qquad \dfrac{a(x-3)}{x^2+25} \to 0$

$\cdot\ x \to 5i : \dfrac{-x^2+2x-31}{x-3} = a + \dfrac{b(x^2+25)}{x-3}$ donc $\underline{a=2}$

$\hookrightarrow \dfrac{-6+10i}{5i-3} = 2 \qquad \dfrac{b(x^2+25)}{x-3} \to 0$

- $\int\left(\frac{2}{x^2+25}-\frac{1}{x-3}\right)dx=\frac{2}{5}\operatorname{Atan}\frac{x}{5}-\ln|x-3|+k$

$\frac{2}{x^2+25}=\frac{2}{25\left(\frac{x^2}{25}+1\right)}=\frac{2}{25\left(\left(\frac{x}{5}\right)^2+1\right)}$ posons $u=\frac{x}{5}$
alors $u'=\frac{1}{5} \Leftrightarrow \frac{2}{5}u'=\frac{2}{25}$
$f=\frac{2}{5}\frac{u'}{u^2+1}$
$F=\frac{2}{5}\operatorname{Atan} u$

66) $\int\cos^2 x\,dx=\int\frac{1+\cos 2x}{2}dx=\int\left(\frac{1}{2}+\underbrace{\frac{1}{2}\cos 2x}\right)dx=\frac{1}{2}x+\frac{1}{4}\sin 2x+k$

$\begin{cases} u=2x \\ u'=2 \Leftrightarrow \frac{1}{4}u'=\frac{1}{2} \\ f=\frac{1}{4}u'\cos u \\ F=\frac{1}{4}\sin u \end{cases}$

67) $\int\cos^3 x\,dx=\frac{1}{12}\sin 3x+\frac{3}{4}\sin x+k$

$\cos 3x=-3\cos x+4\cos^3 x \Leftrightarrow \cos^3 x=\underbrace{\frac{1}{4}\cos 3x}+\frac{3}{4}\cos x$

$u=3x$
$u'=3 \Leftrightarrow \frac{1}{12}u'=\frac{1}{4}$
$f=\frac{1}{12}u'\cos u$
$F=\frac{1}{12}\sin u$

autre méthode :

$\cos^3 x=\cos x\cos^2 x=\cos x\cdot\frac{1}{2}(1+\cos 2x)$
$=\frac{1}{2}\cos x+\frac{1}{2}\cos x\cdot\cos 2x$
$=\frac{1}{2}\cos x+\frac{1}{4}(\cos 3x+\cos x)=\frac{1}{2}\cos x+\frac{1}{4}\cos 3x+\frac{1}{4}\cos x=\frac{3}{4}\cos x+\frac{1}{4}\cos 3x$
etc...

autre méthode :

$\cos^3 x=\cos^2 x\cdot\cos x=(1-\sin^2 x)\cos x=\cos x-\underbrace{\sin^2 x\cos x}$

$u=\sin x$
$u'=\cos x$
$f=u'u^2$
$F=\frac{1}{3}u^3$

$\int\cos^3 x\,dx=\sin x-\frac{1}{3}\sin^3 x+k$

68) $\int\cos^4 x\,dx=\frac{3}{8}x+\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+k$

$\cos^4 x=(\cos^2 x)^2=\left(\frac{1+\cos 2x}{2}\right)^2=\frac{1+2\cos 2x+\cos^2 2x}{4}$
$=\frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{4}\left(\frac{1+\cos 4x}{2}\right)(=\cos^2 2x)$
$=\frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{8}+\frac{1}{8}\cos 4x$
$=\frac{3}{8}+\frac{1}{2}\cos 2x+\frac{1}{8}\cos 4x$

$u=4x$
$u'=4 \Leftrightarrow \frac{1}{32}u'=\frac{1}{8}$
$f=\frac{1}{32}u'\cos u$
$F=\frac{1}{32}\sin u$

69) $\int \sin^4 x \cdot \cos^3 x \, dx = \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + k$

$\sin^4 x \cdot \cos^3 x = \sin^4 x \cdot \cos^2 x \cdot \cos x$
$= \sin^4 x(1-\sin^2 x)\cdot \cos x$
$= \sin^4 x \cdot \cos x - \sin^6 x \cdot \cos x$

$\begin{cases} u = \sin x \\ u' = \cos x \\ f = u^4 \cdot u' - u^6 \cdot u' \\ F = \frac{1}{5}u^5 - \frac{1}{7}u^7 \end{cases}$

70) $\int \sin^7 x \cdot \cos x \, dx = \frac{1}{8}\sin^8 x + k$

$\begin{cases} u = \sin x \\ u' = \cos x \\ f = u^7 \cdot u' \\ F = \frac{1}{8}u^8 + k \end{cases}$

71) $\int \sin x \cdot \cos 2x \, dx = \int \frac{1}{2}(\sin 3x + \sin(-x))\,dx$
$= \frac{1}{2}\int(\sin 3x - \sin x)\,dx$
$= \frac{1}{2}\cdot\left(-\frac{1}{3}\cos 3x + \cos x\right) + k$
$= -\frac{1}{6}\cos 3x + \frac{1}{2}\cos x + k$

$\begin{cases} u = 3x \\ u' = 3 \Leftrightarrow \frac{1}{3}u' = 1 \\ f = \frac{1}{3}u' \sin u \\ F = -\frac{1}{3}\cos u \end{cases}$

72) $\int \cos x \cdot \cos 4x \, dx = \int \frac{1}{2}(\cos 5x + \cos 3x)\,dx$
$= \frac{1}{2}\left(\frac{1}{5}\sin 5x + \frac{1}{3}\sin 3x\right) + k$
$= \frac{1}{10}\sin 5x + \frac{1}{6}\sin 3x + k$

73) $\int \sin 3x \cdot \sin 5x \, dx = \int \frac{1}{2}(\cos 2x - \cos 8x)\,dx$
$= \frac{1}{2}\left(\frac{1}{2}\sin 2x - \frac{1}{8}\sin 8x\right) + k$
$= \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + k$

74) $\int \sin 5x \cdot \cos^3 5x \, dx = -\frac{\cos^4 5x}{20} + k$

$u = \cos 5x$
$u' = -5\sin 5x \Leftrightarrow -\frac{1}{5}u' = \sin 5x$
$f = -\frac{1}{5}u' \cdot u^3$
$F = -\frac{1}{5}\,\frac{1}{4}\cdot u^4 = -\frac{u^4}{20} + k$

75) $\int x \sin 3x \, dx = F(x)$

i.p.p. $\begin{cases} u = x & u' = 1 \\ v' = \sin 3x & v = -\frac{1}{3}\cos 3x \end{cases}$

$F(x) = -\frac{1}{3}x\cos 3x + \frac{1}{3}\int \cos 3x \, dx$

$= \underline{\qquad\qquad} + \frac{1}{3}\cdot\frac{1}{3}\sin 3x + k$

$= -\frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x + k$

76) $\int \ln x \, dx = F(x)$

ipp $\begin{cases} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{cases}$

$F(x) = x\ln x - \int \frac{1}{x}\cdot x \, dx = x\ln x - x + k$

77) $\int x\sqrt{1+2x} \, dx = F(x)$

ipp $\begin{cases} u = x & u' = 1 \\ v' = \sqrt{1+2x} & v = \frac{1}{3}(1+2x)^{3/2} \end{cases}$

$F(x) = \frac{1}{3}x\sqrt{(1+2x)^3} - \frac{1}{3}\int (1+2x)^{3/2} dx$

$= \frac{1}{3}x\cdot\sqrt{(1+2x)^3} - \frac{1}{15}\sqrt{(1+2x)^5} + k$

$u = 1+2x$
$u' = 2 \Leftrightarrow \frac{1}{2}u' = 1$
$f = \frac{1}{2}u'\cdot u^{\frac{1}{2}}$
$F = \frac{1}{2}\cdot\frac{1}{\frac{3}{2}}\cdot u^{\frac{3}{2}}$
$= \frac{1}{3}\sqrt{u^3}$

$u = 1+2x$
$\frac{1}{2}u' = 1$
$f = \frac{1}{2}u'\cdot u^{3/2}$
$F = \frac{1}{2}\,\frac{1}{\frac{5}{2}}\cdot u^{5/2} = \frac{1}{5}u^{\frac{5}{2}}$

78) $\int (7x^2 - 3x + 6)\ln x \, dx = F(x)$

i.p.p. $\begin{cases} u = \ln x & u' = \frac{1}{x} \\ v' = 7x^2 - 3x + 6 & v = \frac{7}{3}x^3 - \frac{3}{2}x^2 + 6x \end{cases}$

$F(x) = \left(\frac{7}{3}x^3 - \frac{3}{2}x^2 + 6x\right)\ln x - \int \frac{1}{x}\cdot\left(\frac{7}{3}x^3 - \frac{3}{2}x^2 + 6x\right) dx$

$= \underline{\qquad\qquad\qquad} - \int\left(\frac{7}{3}x^2 - \frac{3}{2}x + 6\right) dx$

$= \left(\frac{7}{3}x^3 - \frac{3}{2}x^2 + 6x\right)\ln x - \frac{7}{9}x^3 + \frac{3}{4}x^2 - 6x + k$

79) $\int \frac{x}{\cos^2 x}\,dx = F(x)$

i.p.p. $\begin{cases} u = x & u' = 1 \\ v' = \frac{1}{\cos^2 x} & v = \tan x \end{cases}$

$$F(x) = x\cdot\tan x - \int \tan x\,dx$$

$$= \underline{\qquad\qquad} - \int \frac{\sin x}{\cos x}\,dx$$

$$= x\tan x + \ln|\cos x| + k$$

$\begin{cases} u = \cos x \\ u' = -\sin x \Leftrightarrow -u' = \sin x \\ f = -\frac{u'}{u} \\ F = -\ln|u| \end{cases}$

80) $\int x^2 e^{5x}\,dx = F(x)$

i.p.p. $\begin{cases} u = x^2 & u' = 2x \\ v' = e^{5x} & v = \frac{1}{5}e^{5x} \end{cases}$

$$F(x) = \frac{1}{5}x^2e^{5x} - \frac{2}{5}\underbrace{\int xe^{5x}\,dx}_{}$$

i.p.p. $u = x \quad u' = 1$
$v' = e^{5x} \quad v = \frac{1}{5}e^{5x}$

$$F(x) = \frac{1}{5}x^2e^{5x} - \frac{2}{5}\left(\frac{1}{5}xe^{5x} - \frac{1}{5}\int e^{5x}\,dx\right)$$

$$= \underline{\qquad\qquad}\left(\underline{\qquad} - \frac{1}{5}\cdot\frac{1}{5}e^{5x}\right) + k$$

$$= \frac{1}{5}x^2e^{5x} - \frac{2}{25}xe^{5x} + \frac{2}{125}e^{5x} + k$$

$$= \frac{1}{125}e^{5x}\left(25x^2 - 10x + 2\right) + k$$

81) $\int \frac{\ln x}{x^2}\,dx = F(x)$

i.p.p. $\begin{cases} u = \ln x & u' = \frac{1}{x} \\ v' = \frac{1}{x^2} & v = -\frac{1}{x} \end{cases}$

$$F(x) = -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x} + k = -\frac{1+\ln x}{x} + k$$

82) $\int \mathrm{Arcsin}\,x\,dx = F(x)$

i.p.p. $\begin{cases} u = \mathrm{Arcsin}\,x & u' = \frac{1}{\sqrt{1-x^2}} \\ v' = 1 & v = x \end{cases}$

$F(x) = x\,\mathrm{Arcsin}\,x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$= x\,\mathrm{Arcsin}\,x + \sqrt{1-x^2} + k$

$\begin{cases} u = 1-x^2 \\ u' = -2x \Leftrightarrow -\frac{1}{2}u' = x \\ f = -\frac{1}{2}\frac{u'}{u^{\frac{1}{2}}} = -\frac{1}{2}\cdot u' \cdot u^{-\frac{1}{2}} \\ F = -\frac{1}{2}\cdot\frac{1}{\frac{1}{2}}u^{\frac{1}{2}} = -\sqrt{u} \end{cases}$

83) $\int \frac{\mathrm{Arctan}\,x}{1+x^2}\,dx = \frac{1}{2}\mathrm{Arctan}^2 x + k$

posons : $u = \mathrm{Arctan}\,x$

alors : $u' = \frac{1}{1+x^2}$

$f = u \cdot u'$

$F = \frac{1}{2}u^2$

84) $\int \mathrm{Arctan}\,2x\,dx = x\,\mathrm{Arctan}\,2x - \frac{1}{4}\ln|1+4x^2| + k$

i.p.p. $\begin{cases} u = \mathrm{Arctan}\,2x & u' = \frac{2}{1+(2x)^2} = \frac{2}{1+4x^2} \\ v' = 1 & v = x \end{cases}$

$F(x) = x\,\mathrm{Arctan}\,2x - \int \frac{2x}{1+4x^2}\,dx$

$\begin{cases} u = 1+4x^2 \\ u' = 8x \Leftrightarrow \frac{1}{4}u' = 2x \\ f = \frac{1}{4}\frac{u'}{u} \\ F = \frac{1}{4}\ln|u| \end{cases}$

85) $\int \frac{2x-5}{\sqrt{4-x^2}}\,dx = \underbrace{\int \frac{2x}{\sqrt{4-x^2}}\,dx}_{F_1(x)} - \underbrace{\int \frac{5}{\sqrt{4-x^2}}\,dx}_{F_2(x)} = -2\sqrt{4-x^2} - 5\,\mathrm{Arcsin}\,\frac{x}{2} + k$

• calcul de $F_1(x)$:

$u = 4-x^2$

$u' = -2x \Leftrightarrow -u' = 2x$

$f_1 = -\frac{u'}{u^{\frac{1}{2}}} = -u'u^{-\frac{1}{2}}$

$F_1 = -\frac{1}{\frac{1}{2}}\cdot u^{\frac{1}{2}} = -2\sqrt{u}$

$F_1(x) = -2\sqrt{4-x^2}$

• calcul de $F_2(x)$:

$f_2(x) = \frac{5}{\sqrt{4(1-\frac{x^2}{4})}} = \frac{5}{2}\cdot\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$

$u = \frac{x}{2}$

$u' = \frac{1}{2} \Leftrightarrow 5u' = \frac{5}{2}$

$f_2 = 5u'\cdot\frac{1}{\sqrt{1-u^2}}$

$F_2 = 5\cdot\mathrm{Arcsin}\,u$

$F_2(x) = 5\,\mathrm{Arcsin}\,\frac{x}{2}$

86) $\int x(2x+1)^8\,dx = \frac{x}{18}\cdot(2x+1)^9 - \frac{1}{360}(2x+1)^{10} + k$

i.p.p. $\begin{cases} u = x & u' = 1 \\ v' = (2x+1)^8 & v = \frac{1}{18}(2x+1)^9 \end{cases} \longrightarrow \begin{cases} u = 2x+1 \\ u' = 2 \Leftrightarrow \frac{1}{2}u' = 1 \\ f = \frac{1}{2}u'\cdot u^8 \\ F = \frac{1}{2}\cdot\frac{1}{9}u^9 = \frac{u^9}{18} \end{cases}$

$F(x) = \frac{x}{18}(2x+1)^9 - \frac{1}{18}\int(2x+1)^9\,dx \longrightarrow \begin{cases} u = 2x+1 \\ \frac{1}{2}u' = 1 \\ f = \frac{1}{2}u'\cdot u^9 \\ F = \frac{1}{2}\,\frac{1}{10}\cdot u^{10} = \frac{1}{20}u^{10} \end{cases}$

87) $\int \frac{\ln^9 x}{x}\,dx = \frac{1}{10}\ln^{10} x + k$

posons $u = \ln x$
alors $u' = \frac{1}{x}$
$f = u'\cdot u^9$
$F = \frac{1}{10}u^{10}$

88) $\int \frac{\sin 2x}{1+\sin^2 x}\,dx = \ln|1+\sin^2 x| + k$

posons: $u = 1+\sin^2 x$
alors: $u' = 2\sin x\cdot\cos x = \sin 2x$
$f = \frac{u'}{u}$
$F = \ln|u|$

89) $\int \frac{1}{x\ln x}\,dx = \ln|\ln x| + k$

posons: $u = \ln x$
alors: $u' = \frac{1}{x}$
$f = \frac{u'}{u}$
$F = \ln|u|$

90) $\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{2}\int\frac{x^2}{x}\,dx = \frac{1}{2}x^2\ln x - \cancel{\frac{1}{2}\frac{1}{2}}x^2 + k$ $\quad \frac{1}{4}$

i.p.p. $\begin{cases} u = \ln x & u' = \frac{1}{x} \\ v' = x & v = \frac{1}{2}x^2 \end{cases}$

91) $\int x\ln^2 x\,dx = \frac{1}{2}x^2\ln^2 x + \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + k = \frac{1}{4}x^2\left(2\ln^2 x + 2\ln x - 1\right) + k$

i.p.p. $\begin{cases} u = \ln^2 x & u' = 2\ln x \cdot \frac{1}{x} \\ v' = x & v = \frac{1}{2}x^2 \end{cases}$

$F(x) = \frac{1}{2}x^2\ln^2 x + \underbrace{\int x\ln x\,dx}$

i.p.p. $\begin{cases} u = \ln x & u' = \frac{1}{x} \\ v' = x & v = \frac{1}{2}x^2 \end{cases}$

$= \_\_\_\_\_\_ + \frac{1}{2}x^2\ln x - \frac{1}{2}\int \frac{1}{x}x^2\,dx$

$= \_\_\_\_\_\_\_\_\_\_ - \frac{1}{2}\cdot\frac{1}{2}x^2 + k$

92) $\int x^2\ln x\,dx = \frac{1}{3}x^3\ln x - \frac{1}{3}\int \frac{1}{x}x^3\,dx = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + k$

$= \frac{1}{9}x^3\left(3\ln x - 1\right) + k$

i.p.p. $\begin{cases} u = \ln x & u' = \frac{1}{x} \\ v' = x^2 & v = \frac{1}{3}x^3 \end{cases}$

93) $\int \frac{x}{1+x^4}\,dx = \int \frac{x}{1+(x^2)^2} = \frac{1}{2}\,\mathrm{Atan}\,x^2 + k$

posons: $u = x^2$

alors: $u' = 2x \Leftrightarrow \frac{1}{2}u' = x$

$f = \frac{1}{2}\,\frac{u'}{1+u^2}$

$F = \frac{1}{2}\,\mathrm{Atan}\,u + k$

94) $\int \frac{1+\tan^2 x}{\sqrt{1-\tan^2 x}}\,dx = \mathrm{Asin}(\tan x) + k$

Posons: $u = \tan x$

alors: $u' = 1+\tan^2 x$

$f = \frac{u'}{\sqrt{1-u^2}}$

$F = \mathrm{Asin}\,u$

95) $\int \frac{\cos 2x}{1+\sin^2 2x}\,dx = \frac{1}{2}\,\mathrm{Atan}(\sin 2x) + k$

posons: $u = \sin 2x$

alors: $u' = 2\cos 2x \Leftrightarrow \frac{1}{2}u' = \cos 2x$

$f = \frac{1}{2}\,\frac{u'}{1+u^2}$

$F = \frac{1}{2}\,\mathrm{Atan}\,u + k$

96) $\displaystyle\int \frac{3x-7}{1+x^2}\,dx = \underbrace{\int \frac{3x}{1+x^2}\,dx}_{} - 7\int \frac{1}{1+x^2}\,dx$

$\displaystyle = \frac{3}{2}\ln|1+x^2| - 7\cdot \mathrm{Atan}\,x + k$

$\begin{cases} u = 1+x^2 \\ u' = 2x \Leftrightarrow \frac{3}{2}u' = 3x \\ f = \frac{3}{2}\,\frac{u'}{u} \\ F = \frac{3}{2}\ln|u| \end{cases}$

97) $\displaystyle\int x\,\mathrm{Atan}\,x\,dx = \frac{1}{2}x^2\mathrm{Atan}\,x - \frac{1}{2}x + \frac{1}{2}\mathrm{Atan}\,x + k$

i.p.p. $u = \mathrm{Atan}\,x \quad u' = \frac{1}{1+x^2}$

$v' = x \quad v = \frac{1}{2}x^2$

$\displaystyle F(x) = \frac{1}{2}x^2\,\mathrm{Atan}\,x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$

or: $\displaystyle \frac{x^2}{1+x^2} = \frac{x^2+1-1}{1+x^2} = \frac{x^2+1}{1+x^2} - \frac{1}{1+x^2} = 1 - \frac{1}{1+x^2}$

donc $\displaystyle F(x) = \frac{1}{2}x^2\mathrm{Atan}\,x - \frac{1}{2}\left(x - \mathrm{Atan}\,x\right) + k$

98) $\displaystyle\int \frac{\sin 2x - 1}{\cos^2 x}\,dx = \underbrace{\int \frac{\sin 2x}{\cos^2 x}\,dx}_{} - \int \frac{1}{\cos^2 x}\,dx$

$= -\ln|\cos^2 x| - \tan x + k$

$\begin{cases} u = \cos^2 x \\ u' = 2\cos x\cdot(-\sin x) \\ \quad = -\sin 2x \\ f = -\frac{u'}{u} \\ F = -\ln|u| \end{cases}$

99) $\displaystyle\int \frac{2x}{\sin^2 x}\,dx$

i.p.p. $\begin{cases} u = 2x & u' = 2 \\ v' = \frac{1}{\sin^2 x} & v = -\cot x \end{cases}$

(en effet: $\displaystyle \cot' x = \left(\frac{\cos x}{\sin x}\right)' = \frac{-\sin x\sin x - \cos^2 x}{\sin^2 x} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$)

$\displaystyle F(x) = -2x\cot x + 2\int \cot x\,dx$

$\displaystyle = -2x\cot x + 2\cdot\int \frac{\cos x}{\sin x}\,dx$

$= -2x\cot x + 2\ln|\sin x| + k$

$\begin{cases} u = \sin x \\ u' = \cos x \\ f = \frac{u'}{u} \\ F = \ln|u| \end{cases}$