

## Exercice 1

(1) Voir cours.

(2) Soit  $Q(x)$  le polynôme cherché. On a :

$$A(x) + Q(x) = B(x)$$

$$\Leftrightarrow Q(x) = B(x) - A(x)$$

$$\Leftrightarrow Q(x) = -x^3 + \frac{4}{5}x - 1 - (x^3 - 6x^2 - 3x + \frac{2}{7})$$

$$\Leftrightarrow Q(x) = -x^3 + \frac{4}{5}x - 1 - x^3 + 6x^2 + 3x - \frac{2}{7}$$

$$\Leftrightarrow Q(x) = -2x^3 + 6x^2 + \frac{19}{5}x - \frac{9}{7}$$

(3) 1<sup>er</sup> calcul (direct) :

$$\begin{aligned} A(-\sqrt{2}) &= (-\sqrt{2})^3 - 6(-\sqrt{2})^2 - 3(-\sqrt{2}) + \frac{2}{7} \\ &= -2\sqrt{2} - 12 + 3\sqrt{2} + \frac{2}{7} \\ &= \sqrt{2} - \frac{82}{7} \end{aligned}$$

2<sup>e</sup> calcul (par le schéma de Horner):

	1	-6	-3	$\frac{2}{7}$
$-\sqrt{2}$		$-\sqrt{2}$	$6\sqrt{2} + 2$	$-12 + \sqrt{2}$
	1	$-6 - \sqrt{2}$	$6\sqrt{2} - 1$	$\sqrt{2} - \frac{82}{7}$

Donc :  $A(-\sqrt{2}) = \sqrt{2} - \frac{82}{7}$ .

## Exercice 2

Factoriser les expressions suivantes :

(1)  $8x^3 - 64$

$$= 8(x^3 - 8)$$

$$= 8(x - 2)(x^2 + 2x + 4)$$

(2)  $x^2 - 16y^2 - 3ax + 12ay$

$$= (x - 4y)(x + 4y) - 3a(x - 4y)$$

$$= (x - 4y)(x + 4y - 3a)$$

(3)  $(2a - b)^2 - (3a + b)^2$

$$= [(2a - b) - (3a + b)][(2a - b) + (3a + b)]$$

$$= (2a - 3a - b - b)(2a - b + 3a + b)$$

$$= (-a - 2b) \cdot 5a$$

$$= -5a(a + 2b)$$

$$\begin{aligned}
(4) \quad & x^5 - 4x^3 - 8x^2 + 32 \\
&= (x^5 - 4x^3) - (8x^2 - 32) \\
&= x^3(x^2 - 4) - 8(x^2 - 4) \\
&= (x^2 - 4)(x^3 - 8) \\
&= (x - 2)(x + 2)(x - 2)(x^2 + 2x + 4) \\
&= (x - 2)^2(x + 2)(x^2 + 2x + 4) \\
(5) \quad & a^3 - b^3 - (a - b)^3 \\
&= (a - b)(a^2 + ab + b^2) - (a - b)^3 \\
&= (a - b)[(a^2 + ab + b^2) - (a - b)^2] \\
&= (a - b)[a^2 + ab + b^2 - (a^2 - 2ab + b^2)] \\
&= (a - b)[a^2 + ab + b^2 - a^2 + 2ab - b^2] \\
&= 3ab(a - b)
\end{aligned}$$

### Exercice 3

$$\begin{aligned}
(1) \quad & \frac{2x - 11}{5} - \frac{x + 4}{7} = -3 \left( \frac{7}{15} - \frac{3x - 16}{35} \right) \\
\Leftrightarrow & \frac{14x - 77}{35} - \frac{5x + 20}{35} = -\frac{7}{5} + \frac{9x - 48}{35} \\
\Leftrightarrow & \frac{14x - 77}{35} - \frac{5x + 20}{35} = -\frac{49}{35} + \frac{9x - 48}{35} \quad | \cdot 35 \\
\Leftrightarrow & 14x - 77 - 5x - 20 = -49 + 9x - 48 \\
\Leftrightarrow & 9x - 97 = 9x - 97 \\
\Leftrightarrow & 0 = 0
\end{aligned}$$

Donc :  $S = \mathbb{R}$

$$\begin{aligned}
(2) \quad & -2x(x^2 - 4) = x^2 - 2x \\
\Leftrightarrow & -2x(x - 2)(x + 2) = x(x - 2) \\
\Leftrightarrow & -2x(x - 2)(x + 2) - x(x - 2) = 0 \\
\Leftrightarrow & x(x - 2)[-2(x + 2) - 1] = 0 \\
\Leftrightarrow & x(x - 2)(-2x - 5) = 0 \\
\Leftrightarrow & x = 0 \text{ ou } x = 2 \text{ ou } x = -\frac{5}{2}
\end{aligned}$$

Donc :  $S = \{0, 2, -\frac{5}{2}\}$

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